Simulated Clinical Trials

Principe - State of art - Covariate distribution models

Workshop "Modélisation et simulation d'essais cliniques"



Thuesday 09 April 2015

Nicolas SAVY¹, Stéphanie SAVY², Sandrine ANDRIEU²





¹ Mathematics Institute of Toulouse

² INSERM Unit 102

Table of contents



- What is a Simulated Clinical Trial?
- Good Practices for SCT
- Why to simulate Clinical Trials?
- 4 State of art on SCT
- Covariate Distribution Models

Table of contents



- What is a Simulated Clinical Trial?
- 2 Good Practices for SCT
- Why to simulate Clinical Trials?
- State of art on SCT
- Covariate Distribution Models



Make use of the available knowledge about

- drug
- patients
- disease progression
- clinical program

to investigate in silico aspects of the clinical study plan

- dose regimen
- study design
- patients population
- ...

in order to make **rational**, **informed decision** with regards to **optimizing** the development plan of a new compound



Make use of the available knowledge about

- drug
- patients
- disease progression
- clinical program

to investigate in silico aspects of the clinical study plan

- dose regimen
- study design
- patients population
- ...

in order to make **rational**, **informed decision** with regards to **optimizing** the development plan of a new compound



Make use of the available knowledge about

- drug
- patients
- disease progression
- clinical program

to investigate in silico aspects of the clinical study plan

- dose regimen
- study design
- patients population
- ...

in order to make **rational**, **informed decision** with regards to **optimizing** the development plan of a new compound



- Population properties select subjects from the population
 - range of age, of weight, renal function, proportion males / females,...
 - inclusion and exclusion criteria in trial protocol
- Treatment properties specify the relationship patient / treatment
 - nature of treatment for each group (dose size, formulation, frequency,..)
 - kind of treatment assignment (design)
 parallel group, cross-over, forced titration, dose escalation,
 adaptive designs (Talk of Raphael Porcher), SMART design (DTR)
 - Method of treatment assignment: randomization
- Observation properties specify the responses
 - type of responses (biomarker, surrogate or clinical endpoint) to be measured
 - multiple endpoints
 - number and timing of each observations



- Population properties select subjects from the population
 - range of age, of weight, renal function, proportion males / females,...
 - inclusion and exclusion criteria in trial protocol
- Treatment properties specify the relationship patient / treatment
 - nature of treatment for each group (dose size, formulation, frequency,..)
 - kind of treatment assignment (design)
 parallel group, cross-over, forced titration, dose escalation,
 adaptive designs (Talk of Raphael Porcher), SMART design (DTR)
 - Method of treatment assignment : randomization
- Observation properties specify the responses
 - type of responses (biomarker, surrogate or clinical endpoint) to be measured
 - multiple endpoints
 - number and timing of each observations



- Population properties select subjects from the population
 - range of age, of weight, renal function, proportion males / females,...
 - inclusion and exclusion criteria in trial protocol
- Treatment properties specify the relationship patient / treatment
 - nature of treatment for each group (dose size, formulation, frequency,..)
 - kind of treatment assignment (design)
 parallel group, cross-over, forced titration, dose escalation,
 adaptive designs (Talk of Raphael Porcher), SMART design (DTR)
 - Method of treatment assignment: randomization
- Observation properties specify the responses
 - type of responses (biomarker, surrogate or clinical endpoint) to be measured
 - multiple endpoints
 - number and timing of each observations



- Population properties select subjects from the population
 - range of age, of weight, renal function, proportion males / females,...
 - inclusion and exclusion criteria in trial protocol
- Treatment properties specify the relationship patient / treatment
 - nature of treatment for each group (dose size, formulation, frequency,..)
 - kind of treatment assignment (design)
 parallel group, cross-over, forced titration, dose escalation,
 adaptive designs (Talk of Raphael Porcher), SMART design (DTR)
 - Method of treatment assignment : randomization
- Observation properties specify the responses
 - type of responses (biomarker, surrogate or clinical endpoint) to be measured
 - multiple endpoints
 - number and timing of each observations



- previous Phase or previous studies (historical database)
- litterature and / or other databases

```
Meta-Analysis ⇒ Talk of Gilles Chatellier

Database merging ⇒ Talk of Chloé Diméglio

Database sharing ⇒ Talk of Anne-Cambon Thomser
```

- Covariate Distribution Model: How to generate virtual patients?
- PK / PD Models ⇒ Talk of Jérémie Guedj

 Disease progression models (predictive models)
- recruitment dynamic

 Talk of Vladimir Anisimov
 missing data

 Talk of Grégory Guernec
 patients drop-out
 compliance to treatment



- previous Phase or previous studies (historical database)
- litterature and / or other databases

```
Meta-Analysis ⇒ Talk of Gilles Chatellier

Database merging ⇒ Talk of Chloé Diméglio

Database sharing ⇒ Talk of Anne-Cambon Thomsen
```

- Covariate Distribution Model: How to generate virtual patients?
- Input / output Model: How covariates impact the outcomes?
 PK / PD Models => Talk of Jérémie Guedj
 Disease progression models (predictive models)
- Execution Model: What can happens during the C1?
 recruitment dynamic ⇒ Talk of Vladimir Anisimov
 missing data ⇒ Talk of Grégory Guernec
 patients drop-out
 compliance to treatment



- previous Phase or previous studies (historical database)
- litterature and / or other databases

```
Meta-Analysis ⇒ Talk of Gilles Chatellier

Database merging ⇒ Talk of Chloé Diméglio

Database sharing ⇒ Talk of Anne-Cambon Thomsen
```

- Covariate Distribution Model: How to generate virtual patients?
- PK / PD Models ⇒ Talk of Jérémie Guedj

 Disease progression models (predictive models)
- Execution Model: What can happens during the C1's recruitment dynamic ⇒ Talk of Vladimir Anisimov missing data ⇒ Talk of Grégory Guernec patients drop-out compliance to treatment



- previous Phase or previous studies (historical database)
- litterature and / or other databases

```
Meta-Analysis ⇒ Talk of Gilles Chatellier

Database merging ⇒ Talk of Chloé Diméglio

Database sharing ⇒ Talk of Anne-Cambon Thomsen
```

- Covariate Distribution Model: How to generate virtual patients?
- PK / PD Models ⇒ Talk of Jérémie Guedj

 Disease progression models (predictive models)
- Execution Model: What can happens during the CT recruitment dynamic ⇒ Talk of Vladimir Anisimov missing data ⇒ Talk of Grégory Guernec patients drop-out compliance to treatment



- previous Phase or previous studies (historical database)
- litterature and / or other databases

```
Meta-Analysis ⇒ Talk of Gilles Chatellier

Database merging ⇒ Talk of Chloé Diméglio

Database sharing ⇒ Talk of Anne-Cambon Thomsen
```

together with 3 technical machineries

- Covariate Distribution Model: How to generate virtual patients?
- Input / output Model : How covariates impact the outcomes?

```
PK / PD Models ⇒ Talk of Jérémie Guedj
Disease progression models (predictive models)
```

■ Execution Model: What can happens during the CT? recruitment dynamic ⇒ Talk of Vladimir Anisimov missing data ⇒ Talk of Grégory Guernec patients drop-out compliance to treatment



- previous Phase or previous studies (historical database)
- litterature and / or other databases

```
Meta-Analysis ⇒ Talk of Gilles Chatellier

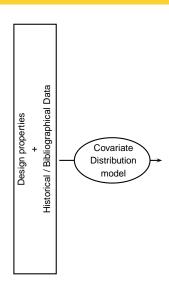
Database merging ⇒ Talk of Chloé Diméglio

Database sharing ⇒ Talk of Anne-Cambon Thomsen
```

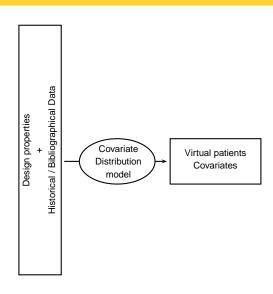
- Covariate Distribution Model: How to generate virtual patients?
- Input / output Model: How covariates impact the outcomes?
 PK / PD Models ⇒ Talk of Jérémie Guedj
 Disease progression models (predictive models)
- Execution Model: What can happens during the CT?
 recruitment dynamic ⇒ Talk of Vladimir Anisimov
 missing data ⇒ Talk of Grégory Guernec
 - missing data \Longrightarrow Talk of Grégory Guel patients drop-out compliance to treatment

Design properties

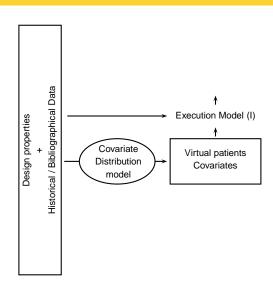
Historical / Bibliographical Data



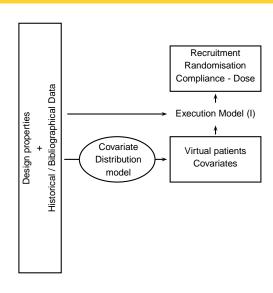




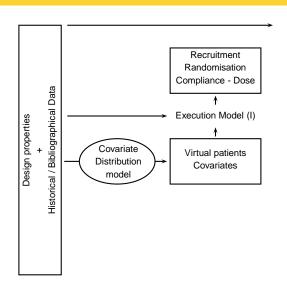




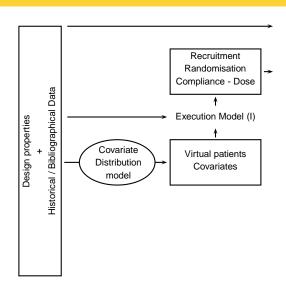




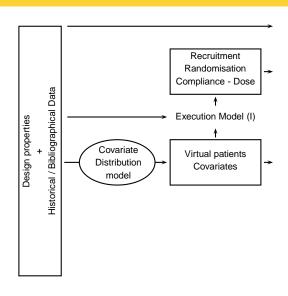




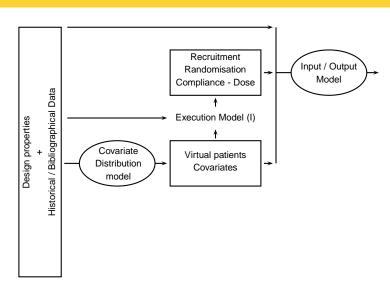




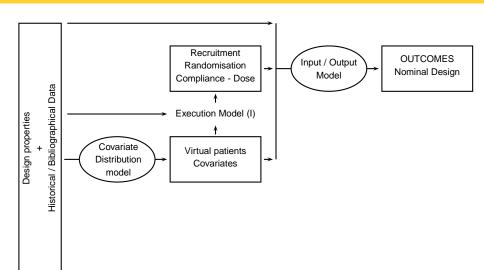




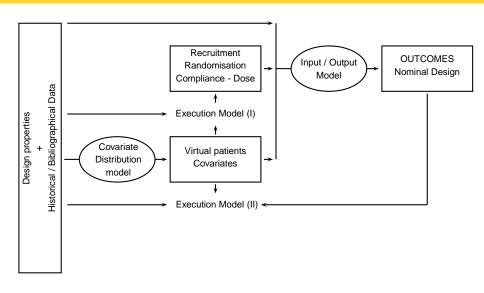




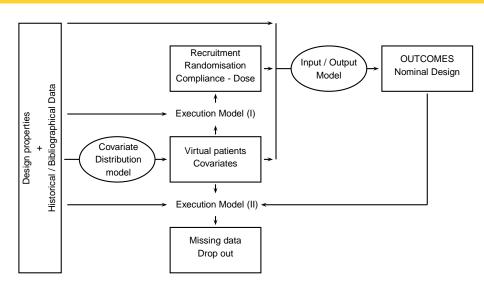














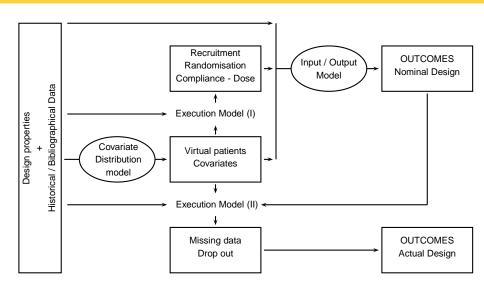


Table of contents



- What is a Simulated Clinical Trial?
- 2 Good Practices for SCT
- Why to simulate Clinical Trials?
- State of art on SCT
- Covariate Distribution Models



Simulation in Drug Development: Good Practices (1999) http://holford.fmbs.auckland.ac.nz/docs/simulation-in-drug-development-good-practices.pdf.

- CLARITY: The report of the simulation should be understandable in terms of scope and conclusions by intended users.
- COMPLETENESS: The assumptions, methods and critical results should be described in sufficient detail to be reproduced by an independent team.
- PARSIMONY: The complexity of the models and simulation procedures should be no more than necessary to meet the objectives of the simulation project.



Simulation in Drug Development: Good Practices (1999) http://holford.fmhs.auckland.ac.nz/docs/simulation-in-drug-development-good-practices.pdf.

- CLARITY: The report of the simulation should be understandable in terms of scope and conclusions by intended users.
- COMPLETENESS: The assumptions, methods and critical results should be described in sufficient detail to be reproduced by an independent team.
- PARSIMONY: The complexity of the models and simulation procedures should be no more than necessary to meet the objectives of the simulation project.



Simulation in Drug Development: Good Practices (1999) http://holford.fmbs.auckland.ac.nz/docs/simulation-in-drug-development-good-practices.pdf.

- CLARITY: The report of the simulation should be understandable in terms of scope and conclusions by intended users.
- COMPLETENESS: The assumptions, methods and critical results should be described in sufficient detail to be reproduced by an independent team.
- PARSIMONY: The complexity of the models and simulation procedures should be no more than necessary to meet the objectives of the simulation project.



Simulation in Drug Development : Good Practices (1999)

http://holford.fmhs.auckland.ac.nz/docs/simulation-in-drug-development-good-practices.pdf.

- Simulation team : composed of
 - an expert in clinical research
 - an expert in statistician and simulation methods
 - a clinical pharmacologist (disease and drug knowledge)
 - eventually a pharmacometrician or an econometrician

has to produce a written document (Simulation plan), with enough detail that another researcher can obtain comparable (simulation)

- Simulation plan : must describe
 - objectives of the SCT
 - assumptions of the models
 - properties and parameters of the trial
 - methods of simulations
 - analyses of the results



Simulation in Drug Development : Good Practices (1999)

http://holford.fmhs.auckland.ac.nz/docs/simulation-in-drug-development-good-practices.pdf.

- Simulation team : composed of
 - an expert in clinical research
 - an expert in statistician and simulation methods
 - a clinical pharmacologist (disease and drug knowledge)
 - eventually a pharmacometrician or an econometrician

has to produce a written document (Simulation plan), with enough detail that another researcher can obtain comparable (simulation)

- Simulation plan : must describe
 - objectives of the SCT
 - assumptions of the models
 - properties and parameters of the trial
 - methods of simulations
 - analyses of the results



Simulation in Drug Development: Good Practices (1999)

http://holford.fmhs.auckland.ac.nz/docs/simulation-in-drug-development-good-practices.pdf.

- Simulation team : composed of
 - an expert in clinical research
 - an expert in statistician and simulation methods
 - a clinical pharmacologist (disease and drug knowledge)
 - eventually a pharmacometrician or an econometrician

has to produce a written document (Simulation plan), with enough detail that another researcher can obtain comparable (simulation)

- Simulation plan : must describe
 - objectives of the SCT
 - assumptions of the models
 - properties and parameters of the trial
 - methods of simulations
 - analyses of the results

Table of contents



- What is a Simulated Clinical Trial?
- 2 Good Practices for SCT
- Why to simulate Clinical Trials?
- State of art on SCT
- Covariate Distribution Models

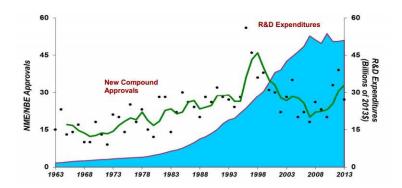


FIGURE: Source: Tufts CSDD; PhRMA, 2014 Industry Profile

- Necessity to develop efficient design / measurement tools ⇒ complex designs
- Necessity to evaluate feasibility of trials

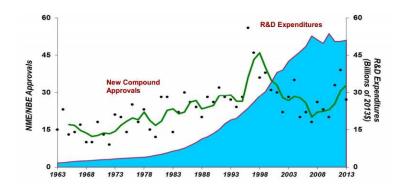


FIGURE: Source: Tufts CSDD; PhRMA, 2014 Industry Profile

- $\bullet \ \ \text{Necessity to develop} \ \textbf{efficient} \ \text{design / measurement tools} \Longrightarrow \text{complex designs}$
- Necessity to evaluate feasibility of trials



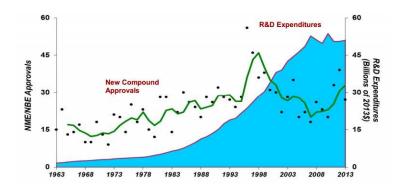


FIGURE: Source: Tufts CSDD; PhRMA, 2014 Industry Profile

- $\bullet \ \ \text{Necessity to develop} \ \textbf{efficient} \ \text{design / measurement tools} \Longrightarrow \text{complex designs}$
- Necessity to evaluate feasibility of trials



- The "negative" trials generate data that contain relevant informations
- see for instance https://www.clinicalstudydatarequest.com/
- Consequently, there is a huge volume of data to explore



- The "negative" trials generate data that contain relevant informations
- The data sharing pulse makes databases easily available
 see for instance https://www.clinicalstudydatarequest.com/
- Consequently, there is a huge volume of data to explore

Data sharing opportunity

- The "negative" trials generate data that contain relevant informations
- The data sharing pulse makes databases easily available
 see for instance https://www.clinicalstudydatarequest.com/
- Consequently, there is a huge volume of data to explore



- Modeling and simulation encouraged in several guidelines
- No guideline on how to do M-S
- Guideline on reporting results of population PK analyses (CHMP/EWP/185990/06) describes expectations of EMA
 - writting of simulation plan
 - writting of a simulation repor
- MSWG in charge of validation



- Modeling and simulation encouraged in several guidelines
- No guideline on how to do M-S
- Guideline on reporting results of population PK analyses (CHMP/EWP/185990/06) describes expectations of EMA
 - writting of simulation plan
 - writting of a simulation repor
- MSWG in charge of validation



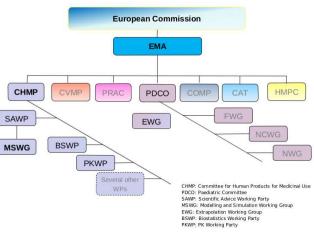
- Modeling and simulation encouraged in several guidelines
- No guideline on how to do M-S
- Guideline on reporting results of population PK analyses (CHMP/EWP/185990/06) describes expectations of EMA
 - writting of simulation plan
 - writting of a simulation report
- MSWG in charge of validation



- Modeling and simulation encouraged in several guidelines
- No guideline on how to do M-S
- Guideline on reporting results of population PK analyses (CHMP/EWP/185990/06) describes expectations of EMA
 - writting of simulation plan
 - writting of a simulation report
- MSWG in charge of validation









EMA (02/06/2014)

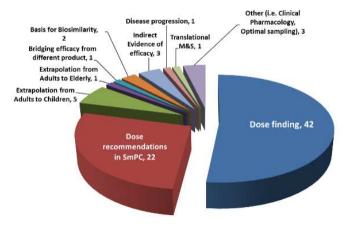


Table of contents



- What is a Simulated Clinical Trial?
- 2 Good Practices for SCT
- Why to simulate Clinical Trials?
- State of art on SCT
- Covariate Distribution Models



- Holford NH, Kimko HC, Monteleone JP, Peck CC Simulation of clinical trials Annu Rev Pharmacol Toxicol. 2000
- Holford NH, Ma SC, Ploeger BA
 Clinical Trials Simulation: a review
 Clinical Pharmacology and Therapeutics. 2010

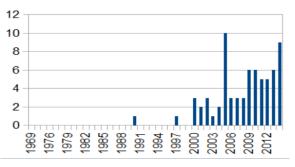


- Holford NH, Kimko HC, Monteleone JP, Peck CC Simulation of clinical trials Annu Rev Pharmacol Toxicol. 2000
- Holford NH, Ma SC, Ploeger BA
 Clinical Trials Simulation: a review
 Clinical Pharmacology and Therapeutics. 2010

Mots clés :	01/01/2000 - 31/01/2010	01/01/2004 - 31/01/2014
« Simulation » « clinical » « trial »	3477	4033
« Trial simulation »	61	70
« Clinical trial simulation »	48	55



- Holford NH, Kimko HC, Monteleone JP, Peck CC Simulation of clinical trials Annu Rev Pharmacol Toxicol. 2000
- Holford NH, Ma SC, Ploeger BA
 Clinical Trials Simulation: a review
 Clinical Pharmacology and Therapeutics. 2010





Nature of these articles

State of art	2	General discussion	6
Methodology	2	Methodology + case study	4
PK / PD	9	Sentivity analysis	3
"Real" simulated clinical trial	29		

Objectives of these articles



Nature of these articles

State of art	2	General discussion	6
Methodology	2	Methodology + case study	4
PK / PD	9	Sentivity analysis	3
"Real" simulated clinical trial	29		

Objectives of these articles

Primary objectives		Secondary objectives	
Virtual partients generation	9		
"Life of the trial"	12	Disease progression	4
		Dropout modelling	6
		Placebo evolution	7
		Protocole deviation	3
Optimizing design	21	Optimizing PK-PD parameters	11
		Optimizing logistic parameters (trial duration,)	4
		Evaluation of the design Robustness	1
		Choice of design properties	8

Table of contents



- What is a Simulated Clinical Trial?
- 2 Good Practices for SCT
- Why to simulate Clinical Trials?
- State of art on SCT
- Covariate Distribution Models

Virtual patients from Bibliographical data



Aim: Create a database of covariate values.

Patients	C ¹	C ²	C_3	C ⁴	 C^k
1	c_1^1	c_1^2	c_1^3	C ₁ ⁴	 c_1^K
2	c_2^1	c_2^2	c_2^3	c_2^4	 c_2^K
n	c_n^1	c_n^2	c_n^3	c_n^4	 c_n^K

① From Bibliography: purely Bayesian approach

- Identify marginal distribution of each C^k
- Generate dataset from these distributions
- Main issue: to take into account correlation between covariates

Virtual patients from Bibliographical data



Aim: Create a database of covariate values.

Patients	C ¹	C ²	C_3	C ⁴	 C^k
1	c_1^1	c_1^2	c_1^3	c_1^4	 c_1^K
2	c_2^1	c_2^2	c_2^3	c_2^4	 c_2^K
n	c_n^1	C_n^2	c_n^3	C_n^4	 c_n^K

① From Bibliography: purely Bayesian approach

- Identify marginal distribution of each C^k
- Generate dataset from these distributions
- Main issue: to take into account correlation between covariates



Assume $X_1 \sim \mathcal{N}(\mu_1, \sigma_1)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2)$ Consider X_1 and X_2 correlated and $\rho = Corr(X_1, X_2)$

- Draw 1000 independant copies of (X₁, X₂)
- Construct
- Consider the 95% IC of Y: [Y₍₂₅₀₎, Y₍₇₅₀₎]

0



Assume $X_1 \sim \mathcal{N}(\mu_1, \sigma_1)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2)$ Consider X_1 and X_2 correlated and $\rho = Corr(X_1, X_2)$

- Draw 1000 independant copies of (X₁, X₂)
- Construct $Y = X_1 + X_2$
- Consider the 95% IC of Y:
 [Y₍₂₅₀₎, Y₍₇₅₀₎]

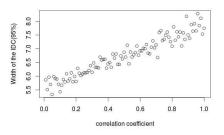
a



- Draw 1000 independent copies of (X₁, X₂)
- Construct $Y = X_1 + X_2$
- Consider the 95% IC of Y: [Y₍₂₅₀₎, Y₍₇₅₀₎]

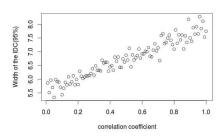


- Draw 1000 independant copies of (X₁, X₂)
- Construct $Y = X_1 + X_2$
- Consider the 95% IC of Y: [Y₍₂₅₀₎, Y₍₇₅₀₎]





- Draw 1000 independant copies of (X₁, X₂)
- Construct $Y = X_1 + X_2$
- Consider the 95% IC of Y: [Y₍₂₅₀₎, Y₍₇₅₀₎]
- Here $\mathbb{E}[Y] = \mathbb{E}[X_1] + \mathbb{E}[X_2]$

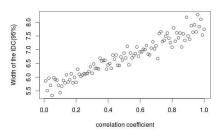




Assume $X_1 \sim \mathcal{N}(\mu_1, \sigma_1)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2)$ Consider X_1 and X_2 correlated and $\rho = Corr(X_1, X_2)$

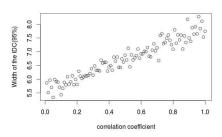
- Draw 1000 independent copies of (X₁, X₂)
- Construct $Y = X_1 \exp(-2X_2)$
- Consider the 95% IC of Y: [Y₍₂₅₀₎, Y₍₇₅₀₎]

•



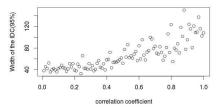


- Draw 1000 independent copies of (X₁, X₂)
- Construct $Y = X_1 \exp(-2X_2)$
- Consider the 95% IC of Y: [Y₍₂₅₀₎, Y₍₇₅₀₎]
- Here $\mathbb{E}[Y] \neq f(\mathbb{E}[X_1], \mathbb{E}[X_2])$





- Draw 1000 independant copies of (X₁, X₂)
- Construct $Y = X_1 \exp(-2X_2)$
- Consider the 95% IC of Y: [Y₍₂₅₀₎, Y₍₇₅₀₎]
- Here $\mathbb{E}[Y] \neq f(\mathbb{E}[X_1], \mathbb{E}[X_2])$



Virtual patients from Historical data



Assume given a database of covariate (for instance a previous C.T.)

Patients	C ¹	C ²	C ³	C ⁴	 C^k
1	c_1^1	c_1^2	c_1^3	C ₁ ⁴	 c_1^k
2	c_2^1	c_2^2	c_2^3	c_2^4	 c_2^k
i	c_i^1	c_i^2	c_i^3	c_i^4	 c_i^k

Aim: to create a **realistic database** from this database.

The links between covariates are (more or less) preserved

Notations: Denote \vec{c}_i the vector of patient *i* covariates.

These covariates can be

- Continuous, in this case, one denotes it ^cC
- Categorial, in this case, one denotes it ^dC

Virtual patients from Historical data



Assume given a database of covariate (for instance a previous C.T.)

Patients	C ¹	C ²	C ³	C ⁴	 C^k
1	c_1^1	c_1^2	c_1^3	C ₁ ⁴	 c_1^k
2	c_2^1	c_2^2	c_2^3	c_2^4	 c_2^k
i	c_i^1	c_i^2	c_i^3	c_i^4	 c_i^k

Aim: to create a realistic database from this database.

The links between covariates are (more or less) preserved

Notations: Denote \vec{c}_i the vector of patient *i* covariates.

These covariates can be

- Continuous, in this case, one denotes it ^cC
- Categorial, in this case, one denotes it ^dC

Virtual patients from Historical data



Assume given a database of covariate (for instance a previous C.T.)

Patients	C ¹	C ²	C ³	C ⁴	 C^k
1	c_1^1	c_1^2	c_1^3	c_1^4	 c_1^k
2	c_2^1	c_2^2	c_2^3	c_2^4	 c_2^k
i	c_i^1	c_i^2	c_i^3	c_i^4	 c_i^k

Aim: to create a realistic database from this database.

The links between covariates are (more or less) preserved

Notations : Denote \vec{c}_i the vector of patient *i* covariates.

These covariates can be:

- Continuous, in this case, one denotes it ^cC
- Categorial, in this case, one denotes it ^dC



② Resampling method (Talk of John O'Quigley)

- Draw \vec{c}_i from the empirical distribution
 - +: data are realistics
 - not allow new configurations of covariates
 May be useful to validate a C.T. design but not to optimize it

3 Marginal Bayesian method

- Fit a distribution for each covariate C^k from historical data
- Draw \vec{c}_i from the fitted distributions
 - +: allow new configurations of covariates
 - correlation structure of covariates is not preserved
 Same issue as the one of bibligraphical based simulation



② Resampling method (Talk of John O'Quigley)

- Draw \vec{c}_i from the empirical distribution
 - +: data are realistics
 - not allow new configurations of covariates
 May be useful to validate a C.T. design but not to optimize it

3 Marginal Bayesian method

- Fit a distribution for each covariate C^k from historical data.
- Draw \vec{c}_i from the fitted distributions
 - +: allow new configurations of covariates
 - correlation structure of covariates is not preserved
 Same issue as the one of bibligraphical based simulation

Virtual patients from Historical data



Tannenbaum SJ, Holford NH, Lee H, Peck CC, Mould DR.
Simulation of correlated continuous and categorical variables using a single multivariate distribution. J Pharmacokinet Pharmacodyn. 2006

Solution

- Fit a distribution for the vector (C^1, C^2, \dots, C^K) from historical data.
- Draw \vec{c}_i from the fitted joined distribution of (C^1, C^2, \dots, C^k) Problem: How to fit such a distribution?

Multinormal Distribution

- If all the covariates are normaly distributed
- It is enough to estimate the mean vector and the variance-covariance matrix
- In most real setting it is a mixed of continuous and categorical distributions

Remark

Problem can be split in groups of independants covariates

 $(\mathbf{C}^1,\dots,\mathbf{C}^L) \text{ independant of } (\mathbf{C}^{L+1},\dots,\mathbf{C}^K) \Longleftrightarrow f_{(\mathbf{C}^1,\dots,\mathbf{C}^K)} = f_{(\mathbf{C}^1,\dots,\mathbf{C}^L)} \times f_{(\mathbf{C}^{L+1},\dots,\mathbf{C}^K)}$

Virtual patients from Historical data



Tannenbaum SJ, Holford NH, Lee H, Peck CC, Mould DR.
Simulation of correlated continuous and categorical variables using a single multivariate distribution. J Pharmacokinet Pharmacodyn. 2006

Solution

- Fit a distribution for the vector (C^1, C^2, \dots, C^k) from historical data.
- Draw \vec{c}_i from the fitted joined distribution of (C^1, C^2, \dots, C^k) Problem: How to fit such a distribution?

4 Multinormal Distribution

- If all the covariates are normaly distributed
- It is enough to estimate the mean vector and the variance-covariance matrix
- In most real setting it is a mixed of continuous and categorical distributions

Remark

Problem can be split in groups of independants covariates

 $(C^1,\ldots,C^L) \text{ independant of } (C^{L+1},\ldots,C^K) \Longleftrightarrow f_{(C^1,\ldots,C^K)} = f_{(C^1,\ldots,C^L)} \times f_{(C^{L+1},\ldots,C^K)}$

Virtual patients from Historical data



Tannenbaum SJ, Holford NH, Lee H, Peck CC, Mould DR. Simulation of correlated continuous and categorical variables using a single multivariate distribution. J Pharmacokinet Pharmacodyn. 2006

Solution

- Fit a distribution for the vector (C^1, C^2, \dots, C^k) from historical data.
- Draw \(\vec{c}_i\) from the fitted joined distribution of \((C^1, C^2, \ldots, C^k\)\)
 Problem: How to fit such a distribution?

4 Multinormal Distribution

- If all the covariates are normaly distributed
- It is enough to estimate the mean vector and the variance-covariance matrix
- In most real setting it is a mixed of continuous and categorical distributions

Remark

Problem can be split in groups of independants covariates

 $(C^1,\dots,C^L) \text{ independant of } (C^{L+1},\dots,C^K) \Longleftrightarrow f_{(C^1,\dots,C^K)} = f_{(C^1,\dots,C^L)} \times f_{(C^{L+1},\dots,C^K)}$

Virtual patients from Historical data



Tannenbaum SJ, Holford NH, Lee H, Peck CC, Mould DR. Simulation of correlated continuous and categorical variables using a single multivariate distribution. J Pharmacokinet Pharmacodyn. 2006

Solution

- Fit a distribution for the vector (C^1, C^2, \dots, C^k) from historical data.
- Draw \vec{c}_i from the fitted joined distribution of (C^1, C^2, \dots, C^k) Problem: How to fit such a distribution?

4 Multinormal Distribution

- If all the covariates are normaly distributed
- It is enough to estimate the mean vector and the variance-covariance matrix
- In most real setting it is a mixed of continuous and categorical distributions

Remark

Problem can be split in groups of independants covariates

$$(C^1,\ldots,C^L)$$
 independant of $(C^{L+1},\ldots,C^K) \Longleftrightarrow f_{(C^1,\ldots,C^K)} = f_{(C^1,\ldots,C^L)} \times f_{(C^{L+1},\ldots,C^K)}$



Idea:

Use **conditional distribution** to separate continuous and categorical covariates

$$f_{(C^1,...,C^K)} = f_{({}^{c}C^1,...,{}^{c}C^L)|({}^{d}C^{L+1},...,{}^{d}C^K)} \times f_{({}^{d}C^{L+1},...,{}^{d}C^K)}$$

Estimation step

- Fit the distribution of $({}^{d}C^{L+1}, \ldots, {}^{d}C^{K})$
- For each configuration $({}^{d}C^{L+1}, \ldots, {}^{d}C^{K})$ fit the distribution of

$$f_{({}^{\circ}C^1,\ldots,{}^{\circ}C^L)|({}^{d}C^{L+1},\ldots,{}^{d}C^K)}$$

If multinormale, compute mean vector and variance-covariance matrix

Simulation step

- Draw a configuration from the fitted multinomiale distribution
- Given this configuration, draw the reminding values from the fitted multinormale distribution



Idea:

Use **conditional distribution** to separate continuous and categorical covariates

$$f_{(C^1,...,C^K)} = f_{({}^{c}C^1,...,{}^{c}C^L)|({}^{d}C^{L+1},...,{}^{d}C^K)} \times f_{({}^{d}C^{L+1},...,{}^{d}C^K)}$$

Estimation step:

- Fit the distribution of $({}^{d}C^{L+1}, \dots, {}^{d}C^{K})$
- For each configuration $({}^dC^{L+1}, \dots, {}^dC^K)$ fit the distribution of

$$f_{(^cC^1,\ldots,^cC^L)|(^dC^{L+1},\ldots,^dC^K)}$$

If multinormale, compute mean vector and variance-covariance matrix

Simulation step

- Draw a configuration from the fitted multinomiale distribution
- Given this configuration, draw the reminding values from the fitted multinormale distribution



Idea:

Use **conditional distribution** to separate continuous and categorical covariates

$$f_{(C^1,...,C^K)} = f_{({}^{c}C^1,...,{}^{c}C^L)|({}^{d}C^{L+1},...,{}^{d}C^K)} \times f_{({}^{d}C^{L+1},...,{}^{d}C^K)}$$

Estimation step:

- Fit the distribution of $({}^{d}C^{L+1}, \dots, {}^{d}C^{K})$
- For each configuration $({}^dC^{L+1}, \dots, {}^dC^K)$ fit the distribution of

$$f_{(^cC^1,\ldots,^cC^L)|(^dC^{L+1},\ldots,^dC^K)}$$

If multinormale, compute mean vector and variance-covariance matrix

Simulation step

- Draw a configuration from the fitted multinomiale distribution
- Given this configuration, draw the reminding values from the fitted multinormale distribution



Idea:

Use **conditional distribution** to separate continuous and categorical covariates

$$f_{(C^1,...,C^K)} = f_{({}^{c}C^1,...,{}^{c}C^L)|({}^{d}C^{L+1},...,{}^{d}C^K)} \times f_{({}^{d}C^{L+1},...,{}^{d}C^K)}$$

Estimation step:

- Fit the distribution of $({}^{d}C^{L+1}, \dots, {}^{d}C^{K})$
- For each configuration $({}^dC^{L+1}, \dots, {}^dC^K)$ fit the distribution of

$$f_{(^cC^1,\ldots,^cC^L)|(^dC^{L+1},\ldots,^dC^K)}$$

If multinormale, compute mean vector and variance-covariance matrix

Simulation step:

- Draw a configuration from the fitted multinomiale distribution
- Given this configuration, draw the reminding values from the fitted multinormale distribution

Virtual patients from Historical data



Assume an HD of 500 patients. The covariates of interest are :

- Sexe (categorical 2 modalities).
- Age (continuous).
- Smoke (categorical 2 modalities (Yes / No)).

Estimation step

- "Fit" the distribution of (Sexe,Smoke)
 - Compute the proportion of each modality
- "Fit" the conditional distribution of Age knowing (Sex,Smoke)
 - Estimate mean and variance under normality assumption

Estimation step: to create SD repeat 500 times

- Draw (Sexe,Smoke) according to 1
- Draw Age according to ②

Remarl

- 4 estimations to estimate the proportions of each of the 4 configurations
- 8 estimations to estimate the parameters of the conditional distributions

Total · 12 estimations



Assume an HD of 500 patients. The covariates of interest are :

- Sexe (categorical 2 modalities).
- Age (continuous).
- Smoke (categorical 2 modalities (Yes / No)).

Estimation step:

- "Fit" the distribution of (Sexe,Smoke)
 - Compute the proportion of each modality
- "Fit" the conditional distribution of Age knowing (Sex,Smoke)
 - Estimate mean and variance under normality assumption

Estimation step: to create SD repeat 500 times:

- Draw (Sexe,Smoke) according to 1
- Draw Age according to ②

Remark

- 4 estimations to estimate the proportions of each of the 4 configurations
- 8 estimations to estimate the parameters of the conditional distributions
 - Total · 12 estimations

Virtual patients from Historical data



Assume an HD of 500 patients. The covariates of interest are :

- Sexe (categorical 2 modalities).
- Age (continuous).
- Smoke (categorical 2 modalities (Yes / No)).

Estimation step:

- "Fit" the distribution of (Sexe,Smoke)
 - Compute the proportion of each modality
- "Fit" the conditional distribution of Age knowing (Sex,Smoke)
 - Estimate mean and variance under normality assumption

stimation step: to create SD repeat 500 times:

- Draw (Sexe,Smoke) according to •
- Draw Age according to ②

Remarl

- 4 estimations to estimate the proportions of each of the 4 configurations
- 8 estimations to estimate the parameters of the conditional distributions

Total: 12 estimations

Virtual patients from Historical data



Assume an HD of 500 patients. The covariates of interest are :

- Sexe (categorical 2 modalities).
- Age (continuous).
- Smoke (categorical 2 modalities (Yes / No)).

Estimation step:

- "Fit" the distribution of (Sexe,Smoke)
 - Compute the proportion of each modality
- "Fit" the conditional distribution of Age knowing (Sex,Smoke)
 - Estimate mean and variance under normality assumption

Estimation step: to create SD repeat 500 times:

- Draw (Sexe,Smoke) according to 0
- Draw Age according to ②

Remark

- 4 estimations to estimate the proportions of each of the 4 configurations
- 8 estimations to estimate the parameters of the conditional distributions

Total : 12 estimations

Virtual patients from Historical data



Assume an HD of 500 patients. The covariates of interest are :

- Sexe (categorical 2 modalities).
- Age (continuous).
- Smoke (categorical 2 modalities (Yes / No)).

Estimation step:

- "Fit" the distribution of (Sexe,Smoke)
 - Compute the proportion of each modality
- "Fit" the conditional distribution of Age knowing (Sex,Smoke)
 - Estimate mean and variance under normality assumption

Estimation step: to create SD repeat 500 times:

- Draw (Sexe,Smoke) according to 0
- Draw Age according to ②

Remark

- 4 estimations to estimate the proportions of each of the 4 configurations
- 8 estimations to estimate the parameters of the conditional distributions

Total: 12 estimations



Assume another HD of 500 patients. The covariates of interest are :

- Weight (continuous)
- Age (continuous)
- BMI (continuous)
- Cholesterol (continuous)
- FBG (continuous)

- D-BP (continuous)
- S-BP (continuous)
- Sex (categorical 2 modalities)
- Smoke (categorical 3 modalities)
- Diagnosis (categorical 4 modalities)

Remark

- 24 estimations to estimate the proportions of each of the 24 configurations
 - 48 estimations to estimate the parameters of the conditional distributions
 Total: 72 estimations
- ② On average, there are $\frac{500}{24} \simeq 21$ values to estimate the multinormal distribution parameters

Virtual patients from Historical data



Assume another HD of 500 patients. The covariates of interest are :

- Weight (continuous)
- Age (continuous)
- BMI (continuous)
- Cholesterol (continuous)
- FBG (continuous)

- D-BP (continuous)
- S-BP (continuous)
- Sex (categorical 2 modalities)
- Smoke (categorical 3 modalities)
- Diagnosis (categorical 4 modalities)

Remark

- U
- 24 estimations to estimate the proportions of each of the 24 configurations
- 48 estimations to estimate the parameters of the conditional distributions
 Total: 72 estimations
- ② On average, there are $\frac{500}{24} \simeq 21$ values to estimate the multinormal distribution parameters



Idea:

Consider all the covariates as continuously distributed (normally or log-normally)

Estimation step

• Estimate the means vector $\vec{\mu}$ and the variance-covariance matrix Σ from the Historical Database

Simulation step

- Draw *n* values $((u_i^1, \ldots, u_i^K)_{i=1,\ldots,n}$ from the multinormal distribution $\mathcal{N}(\vec{\mu}, \Sigma)$
 - For a continuous covariate: finished ${}^{c}c^{k} = u^{k}$ for $k = 1, \dots, L$
 - \bullet For a categorical covariate with M modalities ${}^dC^k$: make use of critical values

$$CrV_{m}^{k} = \mu_{k} + \Sigma_{k,k}\phi^{-1}(\sum_{i=1}^{m} p_{i}), \qquad 1 \leq m \leq M$$

- $(p_m: 1 \le m \le M)$ the proportions of each modality estimated from H.D.
- μ_k and $\Sigma_{k,k}$ the parameters of the normal distribution fitted from H.D.
- d denote the standard normal distribution



Idea:

Consider all the covariates as continuously distributed (normally or log-normally)

Estimation step:

ullet Estimate the means vector $ec{\mu}$ and the variance-covariance matrix Σ from the Historical Database

Simulation step

- Draw *n* values $((u_i^1, \dots, u_i^K)_{i=1,\dots,n}$ from the multinormal distribution $\mathcal{N}(\vec{\mu}, \Sigma)$
 - For a **continuous covariate**: finished ${}^{c}c^{k} = u^{k}$ for k = 1, ..., L
 - \bullet For a categorical covariate with M modalities ${}^dC^k$: make use of critical values

$$CrV_m^k = \mu_k + \Sigma_{k,k}\phi^{-1}(\sum_{i=1}^m p_i), \qquad 1 \le m \le M$$

- $(p_m; 1 < m < M)$ the proportions of each modality estimated from H.D.
- μ_k and $\Sigma_{k,k}$ the parameters of the normal distribution fitted from H.D.
- φ denote the standard normal distribution



Idea:

Consider all the covariates as continuously distributed (normally or log-normally)

Estimation step:

ullet Estimate the means vector $ec{\mu}$ and the variance-covariance matrix Σ from the Historical Database

Simulation step:

- Draw *n* values $((u_i^1, \dots, u_i^K)_{i=1,\dots,n}$ from the multinormal distribution $\mathcal{N}(\vec{\mu}, \Sigma)$
 - For a continuous covariate: finished ${}^{c}c^{k}=u^{k}$ for $k=1,\ldots,L$
 - For a categorical covariate with M modalities ^dC^k: make use of critical values

$$CrV_m^k = \mu_k + \Sigma_{k,k}\phi^{-1}(\sum_{i=1}^m p_i), \qquad 1 \le m \le M$$

- $(p_m; 1 < m < M)$ the proportions of each modality estimated from H.D.
- μ_k and $\Sigma_{k,k}$ the parameters of the normal distribution fitted from H.D.
- φ denote the standard normal distribution



Idea:

Consider all the covariates as continuously distributed (normally or log-normally)

Estimation step:

ullet Estimate the means vector $\vec{\mu}$ and the variance-covariance matrix Σ from the Historical Database

Simulation step:

- Draw *n* values $((u_i^1, \dots, u_i^K)_{i=1,\dots,n}$ from the multinormal distribution $\mathcal{N}(\vec{\mu}, \Sigma)$
 - For a continuous covariate: finished ${}^{c}c^{k}=u^{k}$ for $k=1,\ldots,L$
 - For a categorical covariate with M modalities a CK: make use of critical values

$$CrV_m^k = \mu_k + \Sigma_{k,k}\phi^{-1}(\sum_{i=1}^m p_i), \qquad 1 \le m \le M$$

- $(p_m; 1 < m < M)$ the proportions of each modality estimated from H.D.
- μ_k and $\Sigma_{k,k}$ the parameters of the normal distribution fitted from H.D.
- φ denote the standard normal distribution



Idea:

Consider all the covariates as continuously distributed (normally or log-normally)

Estimation step:

ullet Estimate the means vector $ec{\mu}$ and the variance-covariance matrix Σ from the Historical Database

Simulation step:

- Draw *n* values $((u_i^1, \ldots, u_i^K)_{i=1,\ldots,n}$ from the multinormal distribution $\mathcal{N}(\vec{\mu}, \Sigma)$
 - For a continuous covariate: finished ${}^{c}c^{k}=u^{k}$ for $k=1,\ldots,L$
 - ullet For a categorical covariate with M modalities ${}^dC^k$: make use of critical values

$$CrV_m^k = \mu_k + \Sigma_{k,k}\phi^{-1}(\sum_{i=1}^m p_i), \quad 1 \le m \le M$$

- $(p_m; 1 < m < M)$ the proportions of each modality estimated from H.D.
- μ_k and $\Sigma_{k,k}$ the parameters of the normal distribution fitted from H.D.
- φ denote the standard normal distribution



Idea:

Consider all the covariates as continuously distributed (normally or log-normally)

Estimation step:

ullet Estimate the means vector $\vec{\mu}$ and the variance-covariance matrix Σ from the Historical Database

Simulation step:

- Draw *n* values $((u_i^1, \dots, u_i^K)_{i=1,\dots,n}$ from the multinormal distribution $\mathcal{N}(\vec{\mu}, \Sigma)$
 - For a continuous covariate: finished ${}^{c}c^{k}=u^{k}$ for $k=1,\ldots,L$
 - \bullet For a categorical covariate with M modalities ${}^dC^k$: make use of critical values

$$CrV_m^k = \mu_k + \Sigma_{k,k}\phi^{-1}(\sum_{i=1}^m p_i), \qquad 1 \leq m \leq M$$

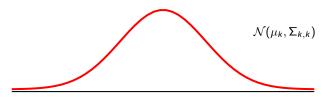
- $(p_m; 1 \le m \le M)$ the proportions of each modality estimated from H.D.
- μ_k and $\Sigma_{k,k}$ the parameters of the normal distribution fitted from H.D.
- \bullet denote the standard normal distribution.

Virtual patients from Historical data



Assume M equal to 3 modalities.

Modality	${}^dC^k=1$	$^{d}C^{k}=2$	$^{d}C^{k}=3$
Proportion	p_1	p_2	<i>p</i> ₃

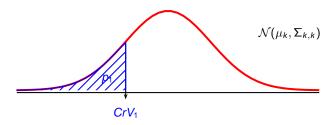


Virtual patients from Historical data



Assume *M* equal to 3 modalities.

Modality	${}^{d}C^{k}=1$	$^{d}C^{k}=2$	$^{d}C^{k}=3$
Proportion	p_1	p_2	<i>p</i> ₃



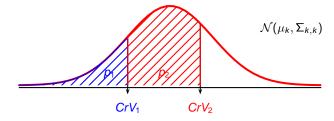
For patient *i*, we have

$$-\infty < u_i^k \le CrV_1^k$$
 then ${}^dC_i^k := 1$



Assume *M* equal to 3 modalities.

Modality	${}^dC^k=1$	$^{d}C^{k}=2$	$^{d}C^{k}=3$
Proportion	p_1	p_2	<i>p</i> ₃



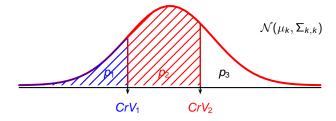
For patient i, we have

$$-\infty < u_i^k \le CrV_1^k$$
 then ${}^dC_i^k := 1$
 $CrV_1^k < u_i^k \le CrV_2^k$ then ${}^dC_i^k := 2$



Assume *M* equal to 3 modalities.

Modality	${}^{d}C^{k}=1$	$^{d}C^{k}=2$	$^{d}C^{k}=3$
Proportion	p_1	p_2	<i>p</i> ₃



For patient i, we have

$$-\infty < u_i^k \le CrV_1^k$$
 then ${}^dC_i^k := 1$
 $CrV_1^k < u_i^k \le CrV_2^k$ then ${}^dC_i^k := 2$
 $CrV_2^k < u_i^k \le +\infty$ then ${}^dC_i^k := 3$



Simulation study in Tannenbaum et al.

- 1 categorical distribution with 2 modalities
- 2 continuous distributions CONT1 and CONT2 log-normaly distributed

Parameters of the simulation scenarios



Simulation study in Tannenbaum et al.

- 1 categorical distribution with 2 modalities
- 2 continuous distributions CONT1 and CONT2 log-normaly distributed

Parameter	CONT1			CONT2
Mean (CAT = 1)	10 50 90			90
Mean (CAT = 2)	100			100
Mode ratio	0.1 0.5 0.9		0.9	
CV (%)	30		30	
Minimum	0			0
Maximum	1000		1000	

Parameters of the simulation scenarios



Simulation study in Tannenbaum et al.

- 1 categorical distribution with 2 modalities
- 2 continuous distributions CONT1 and CONT2 log-normaly distributed

Parameter	CONT1			CONT2
Mean (CAT = 1)	10 50 90			90
Mean (CAT = 2)	100			100
Mode ratio	0.1 0.5 0.9			0.9
CV (%)	30		30	
Minimum	0			0
Maximum	1000			1000

Parameters of the simulation scenarios

Parameter	values		
Mode ratio	0.1	0.5	0.9
CORR between CONT1 and CONT2	0	0.45	0.9
% of (CAT = 1)	10	25	50



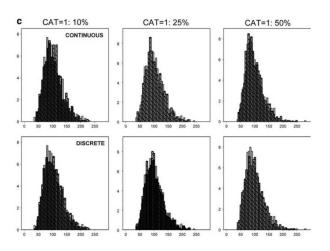


FIGURE: Marginal distribution of CONT1 for MR = 0.9 and R = 0



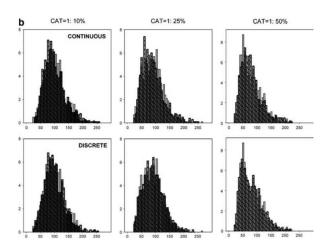


FIGURE: Marginal distribution of CONT1 for MR = 0.5 and R = 0



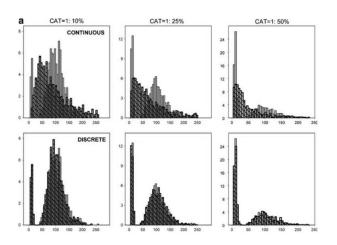
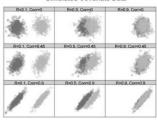


FIGURE: Marginal distribution of CONT1 for MR = 0.1 and R = 0

Virtual patients from Historical data



Simulated Covariate Data



| Discrete Method | Red S, Corrid | Red S, Corrid | Red S, Corrid | Red S, Corrid S | Red S, Corrid S

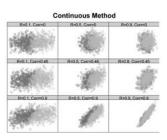


FIGURE: Correlation between CONT1 and CONT2 with 50% of patients in each subgroup



Thank you for your attention...



Thank you for your attention...



Do not be scared,

It is a SCT ...