

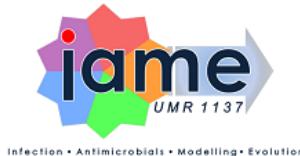
Colloque du club SMAC – Cancéropôle GSO  
« Statistiques et mathématiques appliquées au cancer »  
Modélisation et simulation d'essais cliniques

# AN INTRODUCTION TO MODELS AND METHODS IN PK-PD

**ADRIEN TESSIER**

*adrien.tessier@inserm.fr*

IAME, INSERM UMR 1137, University Paris Diderot, Sorbonne Paris Cité

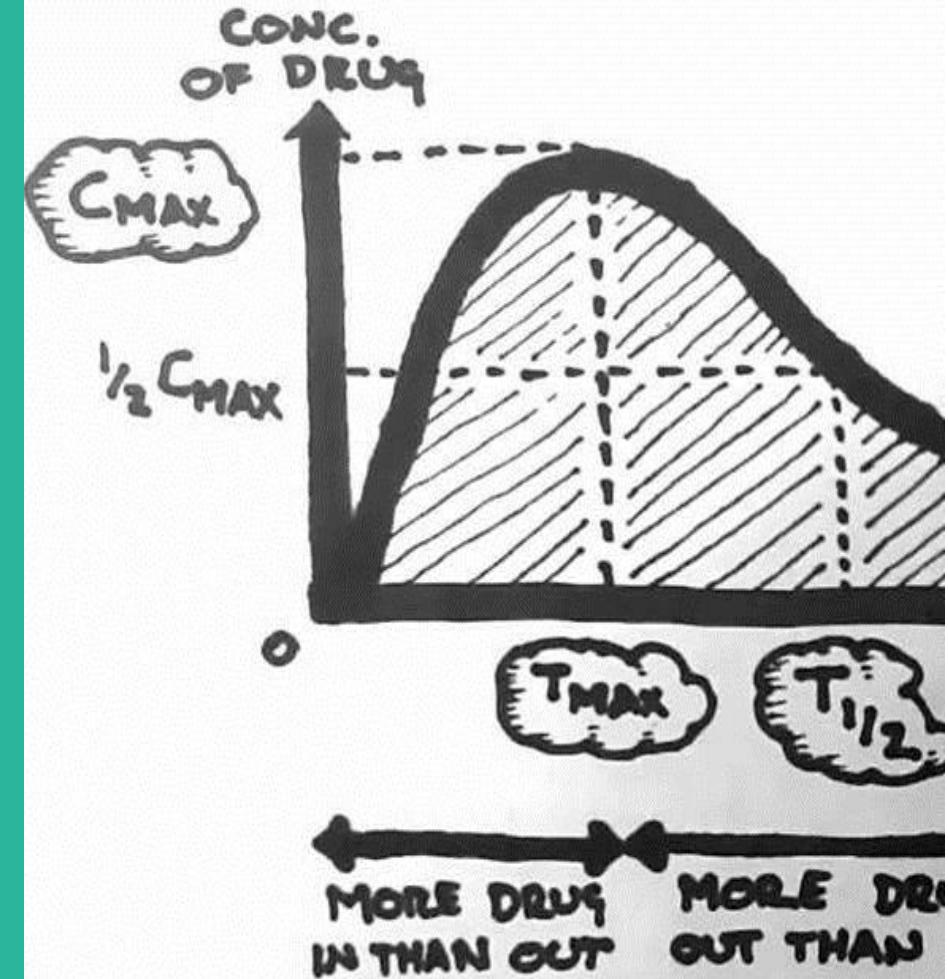


April 2015 - Toulouse, Institut de Mathématiques

# O U T L I N E

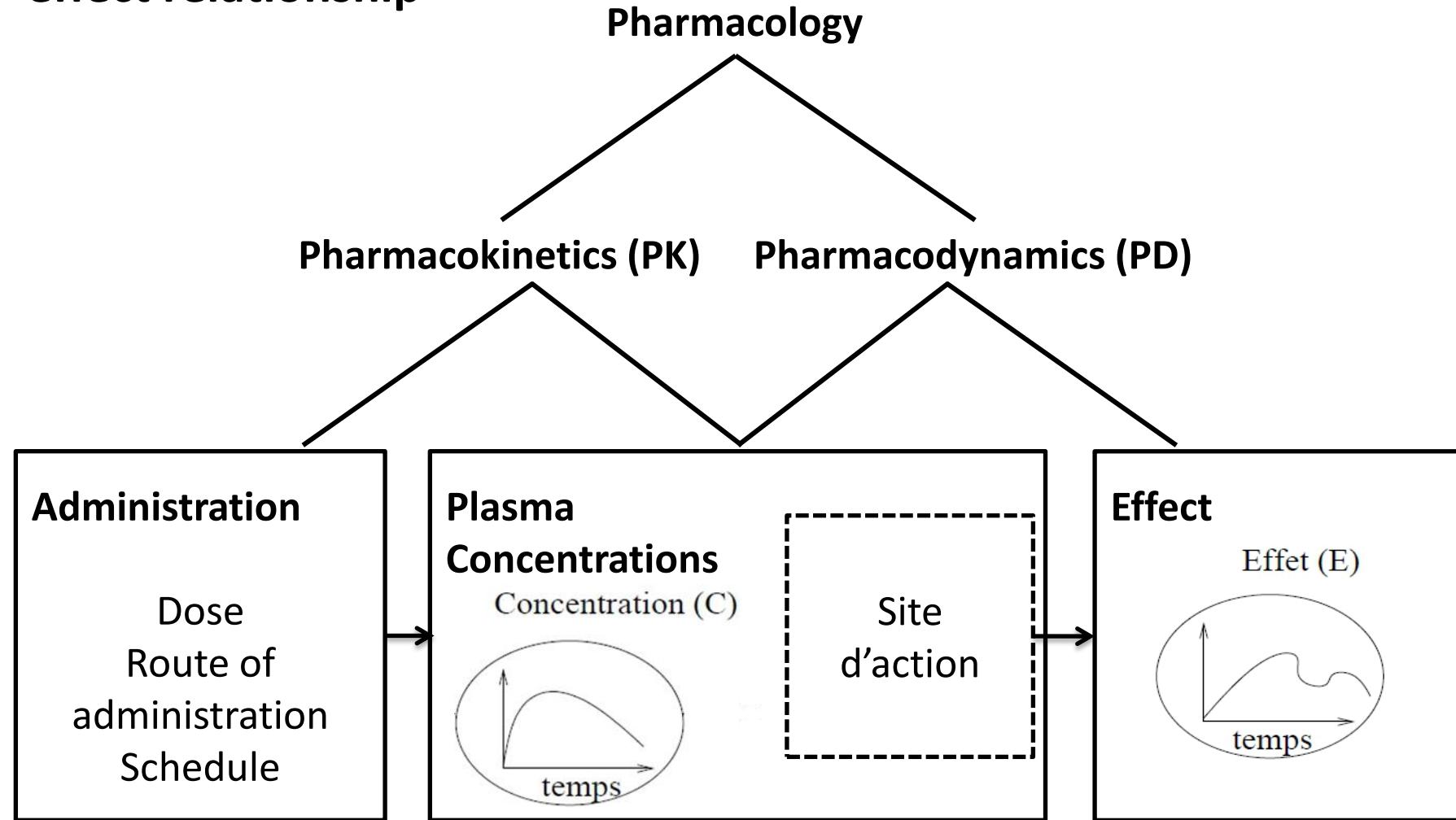
- 1. Models in pharmacokinetics and pharmacodynamics**
- 2. Statistical methods**  
**Pharmacometrics**
- 3. Treatment individualization**
- 4. Design optimization**
- 5. Conclusion**

# MODELS IN PHARMACOKINETICS AND PHARMACODYNAMICS



# PHARMACOKINETICS & PHARMACODYNAMICS

## Dose - effect relationship



# PHARMACOKINETICS & PHARMACODYNAMICS

**Pharmacokinetics (PK) : « What the body does to the drug»**

- Descriptive and quantitative study of the fate of substances in the body
  - drug concentrations over time

**Pharmacodynamics (PD) : « What the drug does to the body or the pathogen»**

- relationship between drug concentration and effect of the drug
- the effect of the drug depends on its concentration on the site of action
- generally the blood is considered as a reflect of the drug concentration on the site of action
- this is why it is critical to know what drives this concentration
- Variety of markers depending on the context
  - biological markers, pathogen concentration (viral load)
  - clinical markers (pain)
  - continuous, discrete, categorical

# MODELS IN PHARMACOKINETICS AND PHARMACODYNAMICS

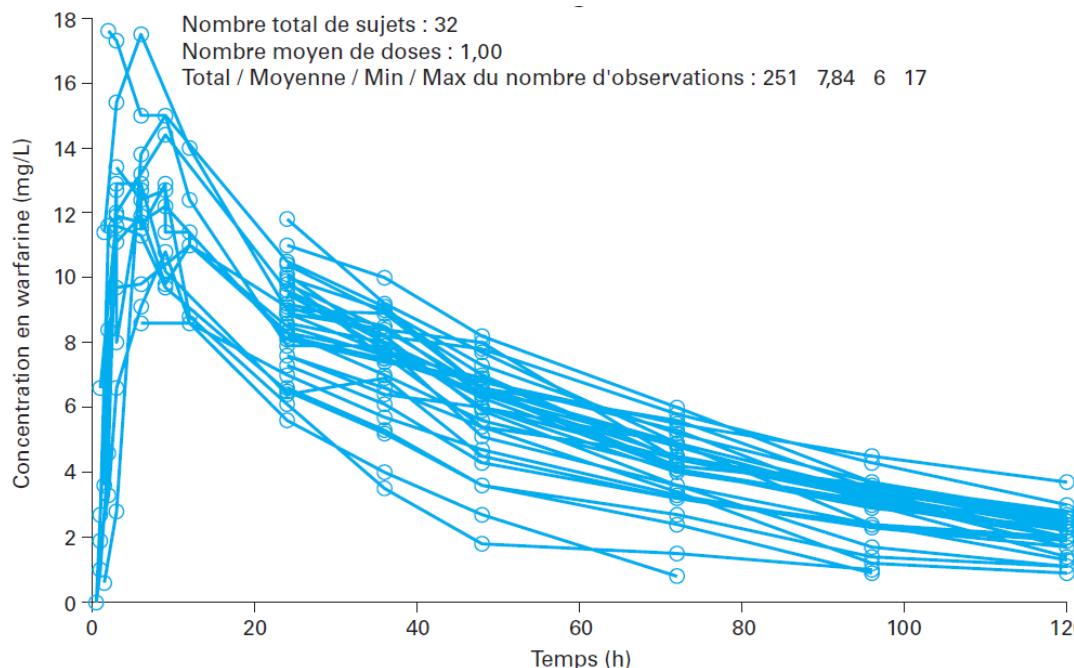
## TYPICAL DATA IN PHARMACOKINETICS

6

### Warfarin: anticoagulant

32 healthy volunteers

- PK data : plasma concentration after a unique oral administration
- objective: characterization of a median profile and the **between-subjects variability**



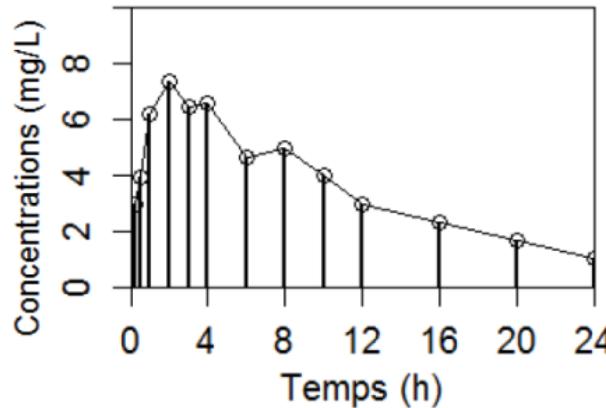
Holford, N. H. Clinical pharmacokinetics and pharmacodynamics of warfarin. Understanding the dose-effect relationship. *Clin. Pharmacokinet.* **11**, 483–504 (1986).

# MODELS IN PHARMACOKINETICS AND PHARMACODYNAMICS

## ANALYSIS OF PHARMACOKINETIC DATA

7

### Non-compartmental analysis



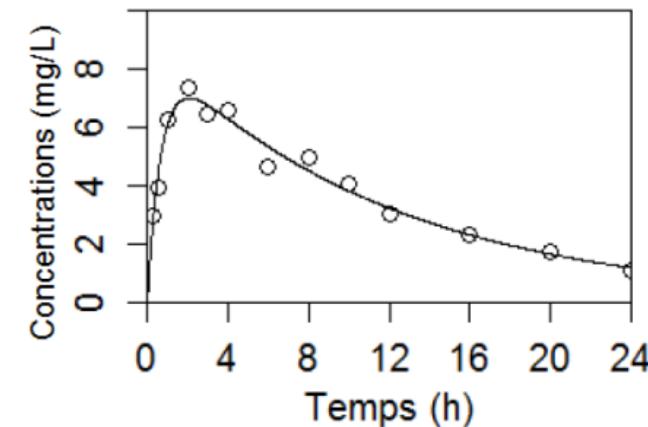
Estimate parameters summarizing the PK profile:

- area under the curve (AUC)
- maximum Concentration ( $C_{\max}$ )
- terminal half-life ( $t_{1/2}$ )

Directly on the observed concentrations:

- few assumptions
- > 10 concentrations per subject
- calculation using trapezoidal method (linear or log-linear)

### Modeling



Model the **whole course** of drug concentrations  
The body is considered as a set of compartments

- homogeneous kinetics in a compartment
- transfers between the compartments
- requires to understand the main determinants of the drug PK

The transfers between the compartments are modeled using **differential equations**:

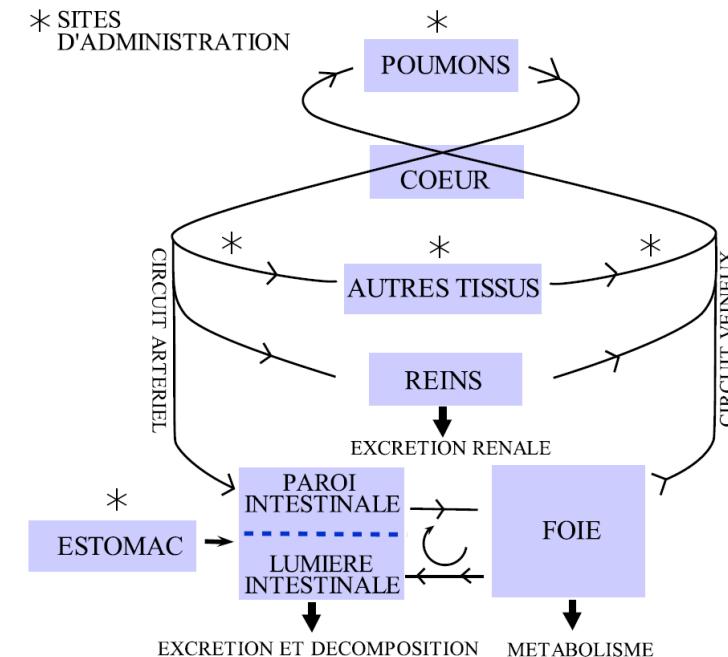
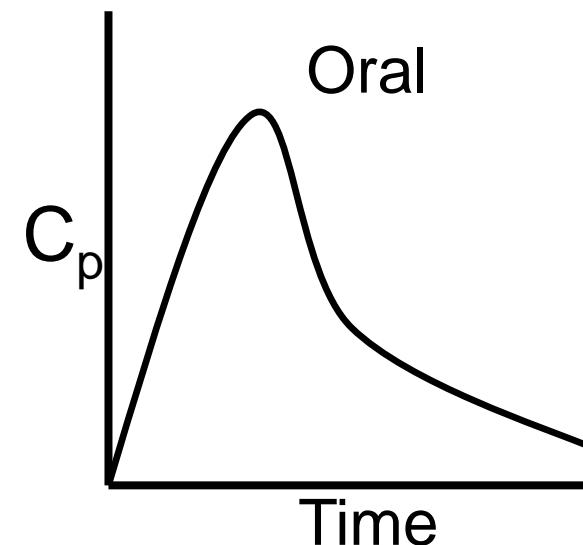
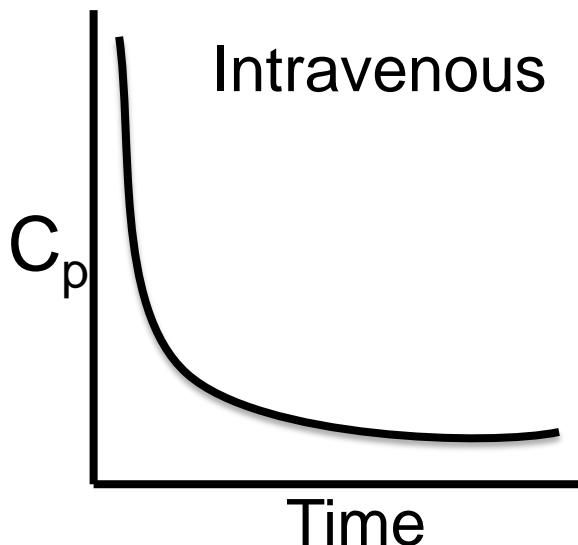
- parameters have a biological meaning
- models are **non-linear**

# PK DETERMINANTS: ADME PROCESS

## Absorption

Transfer of the drug from the site of administration to the blood

The route of administration will impact on:



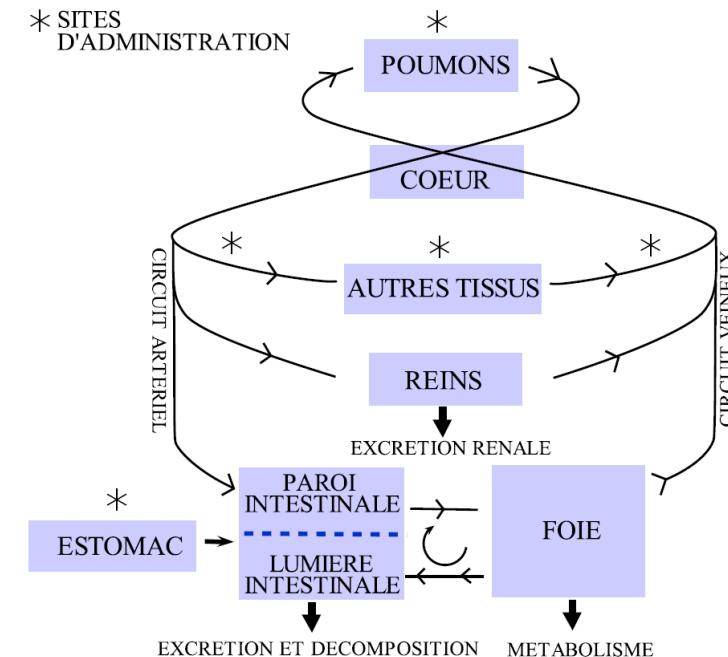
- The fraction of the dose reaching in the circulation ( $F$  = bioavailability)
- The time to reach the circulation (absorption)

# PK DETERMINANTS: ADME PROCESS

## Distribution

Diffusion of the drug through the blood in the different organs and tissues

Volume of distribution (V): theoretical volume that a drug would have to occupy to provide the same concentration as it currently is in blood plasma



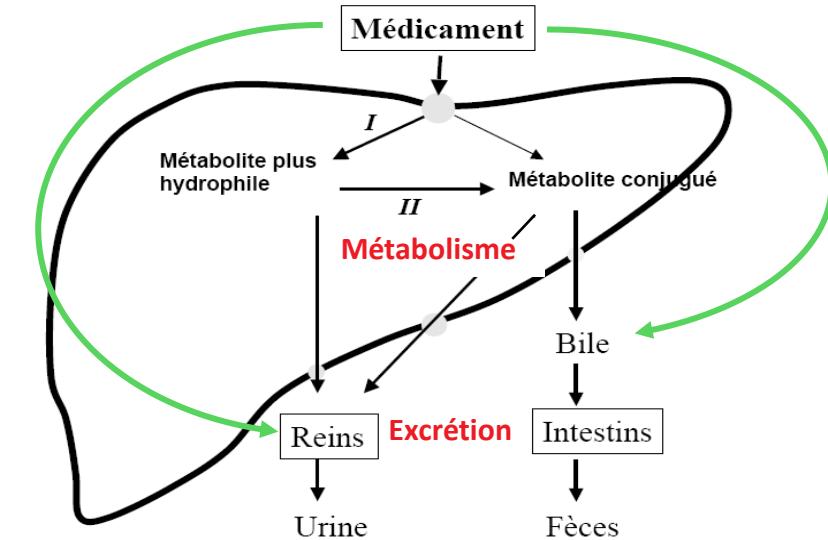
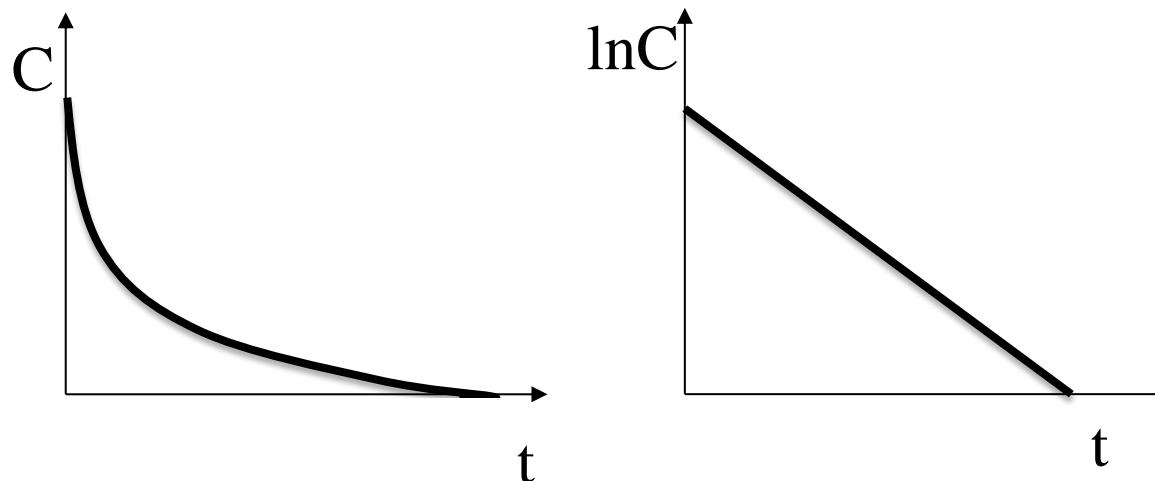
$$V = \frac{\text{Total amount of drug reaching the circulation}}{\text{Plasma concentration}} = \frac{A}{C}$$

# PK DETERMINANTS: ADME PROCESS

## Elimination

**Metabolism** (transformation of the drug)

**Excretion** (elimination of the drug)



- Generally the elimination is a first-order process
  - $C(t) = C(0)e^{-kt}$

# PK DETERMINANTS: ADME PROCESS

## Elimination

**Metabolism** (transformation of the drug)

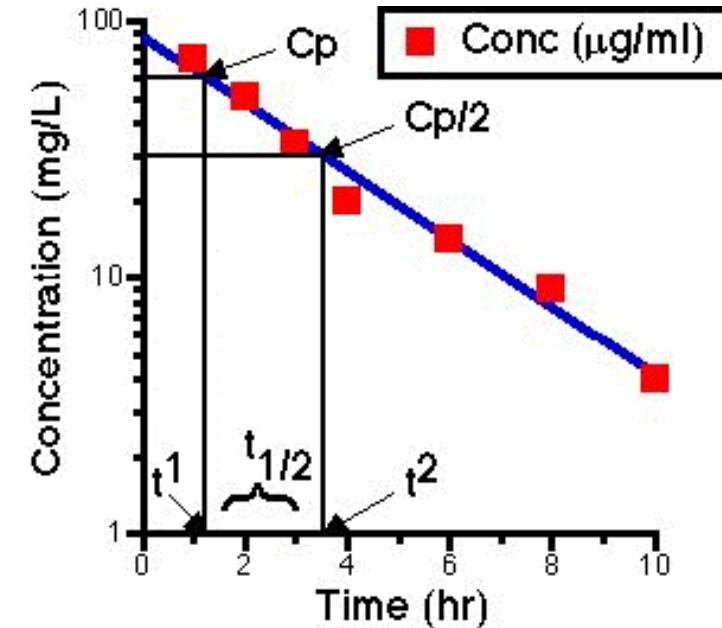
**Excretion** (elimination of the drug)

- Generally the elimination is a first-order process

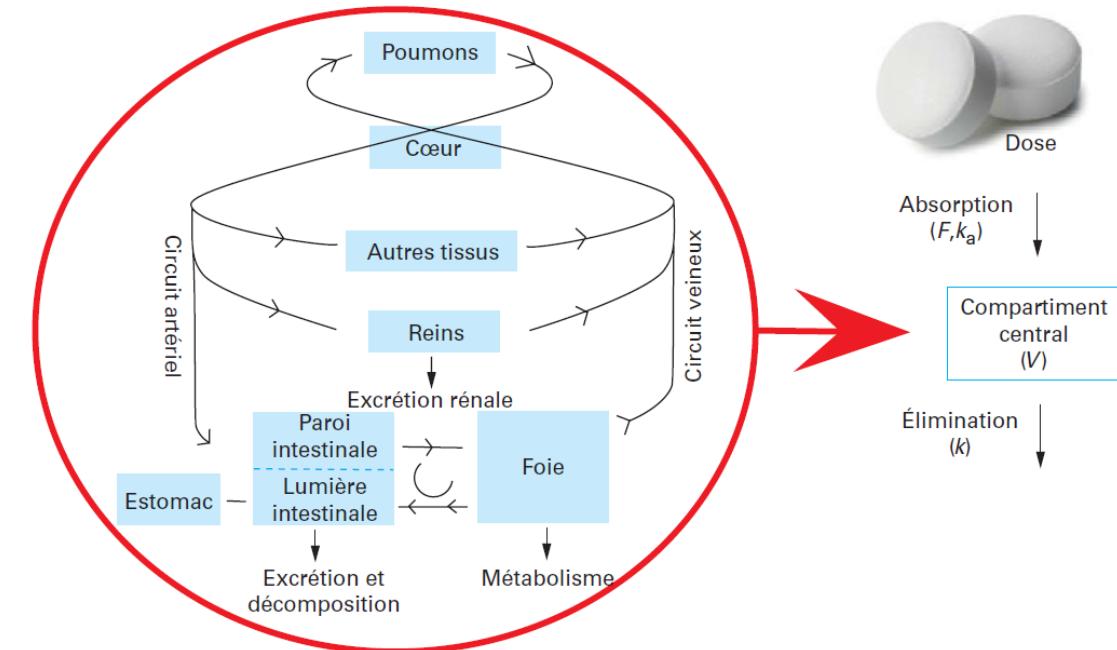
$$\frac{dC}{dt} = -k \cdot C$$

$t_{1/2} = \frac{\ln(2)}{k}$  the half life : time to reduce the amount of drug by 50%

$Cl = k \cdot V$  the drug clearance : the volume of blood cleared per unit of time



# PHARMACOKINETIC MODEL



**Differential equations = mass balance**

$$\frac{dA(1)}{dt} = -ka \times A(1)$$

$$A(1)_{t=0} = F \times \text{Dose}$$

$$\frac{dA(2)}{dt} = ka \times A(1) - k \times A(2)$$

$$A(2)_{t=0} = 0$$

$$k = CL/V$$

$A(1)$ : drug quantity in depot compartment (gut)

$A(2)$ : drug quantity in central compartment (measure compartment)

- observed concentration  $C(t) = A(2)/V$

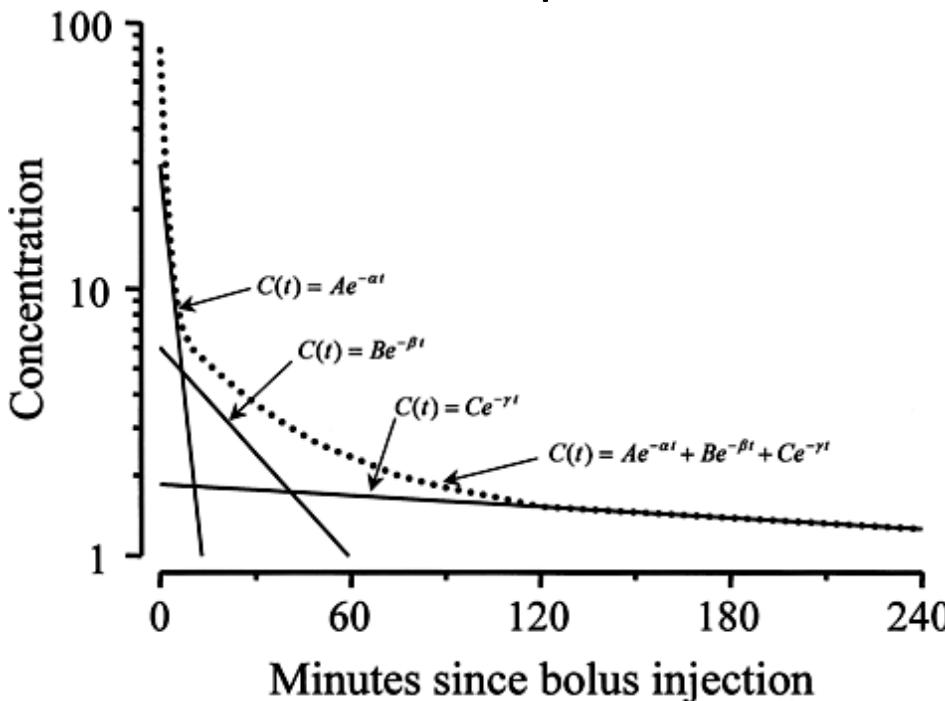
**Analytical solution (Laplace transformation) :**

$$C(t) = \frac{F \times \text{Dose}}{V} \frac{ka}{(ka - \frac{CL}{V})} \left( e^{-\frac{CL}{V}t} - e^{-ka t} \right)$$

# PHARMACOKINETIC MODEL

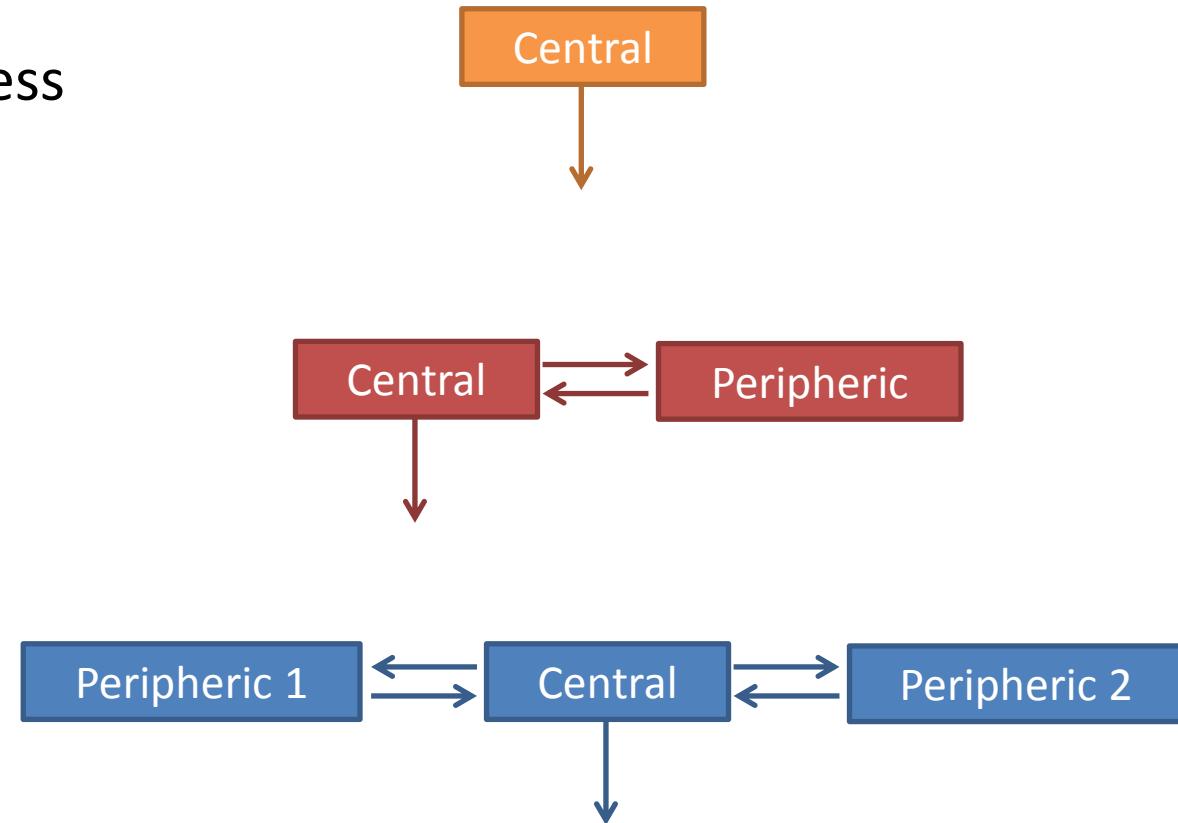
## Empirical models

- Simplification of the ADME process
  - 1 to 3 compartments



PK profile represented on log-scale

- Number of decreasing slopes = number of compartments



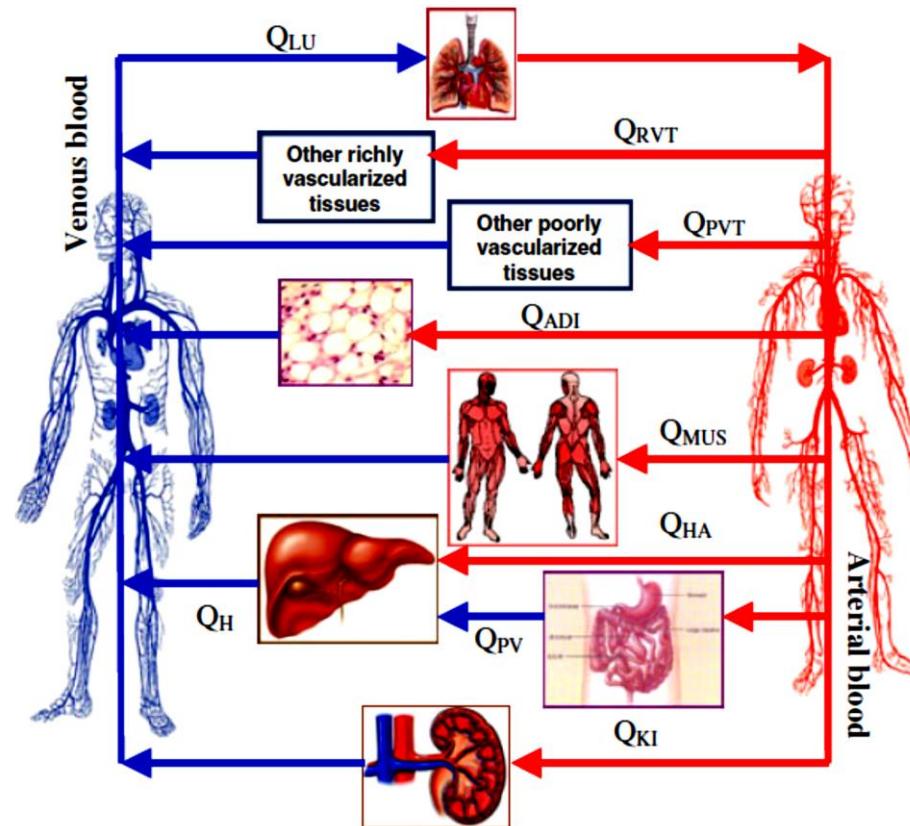
# MODELS IN PHARMACOKINETICS AND PHARMACODYNAMICS

## PHARMACOKINETIC MODEL

14

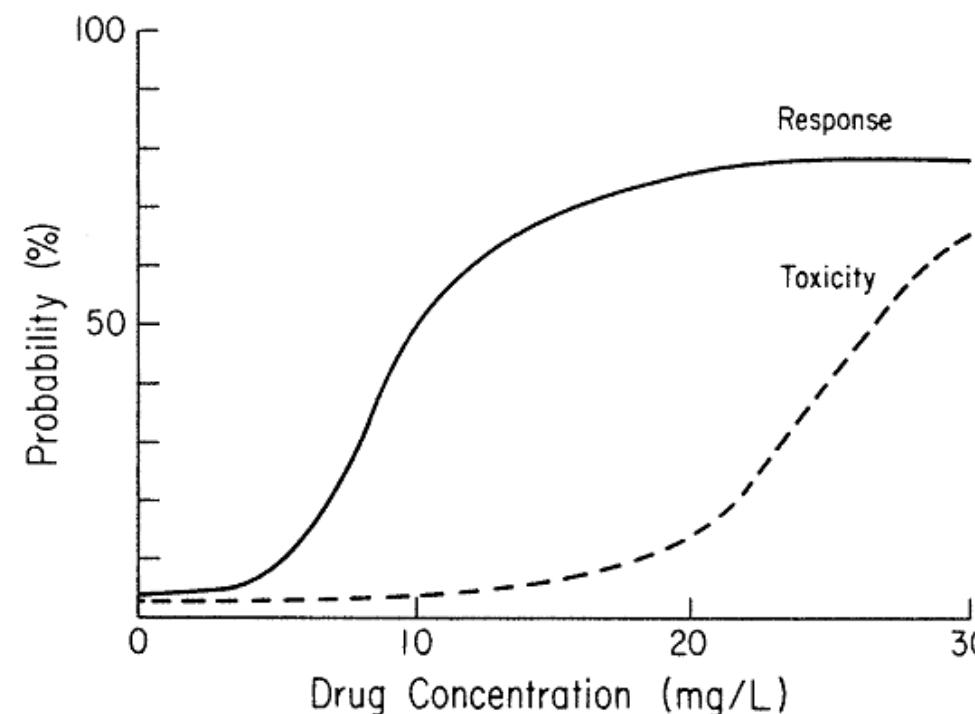
### Physiological models

- PBPK : Physiologically Based Pharmacokinetics
  - using biologic and *in vitro* data



# PHARMACODYNAMIC MODEL

- Direct or indirect relationship between drug concentrations and effect
- A too high drug exposure increases the risk of toxicity
- Purpose : find the best therapeutic window, *i.e.* a balance between drug efficacy and toxicity



# PHARMACODYNAMIC MODEL

## Direct response model

Direct relationship between drug concentrations and effect

$$E(t) = E_0 \left( 1 + E_{max} \frac{C}{C + C_{50}} \right)$$

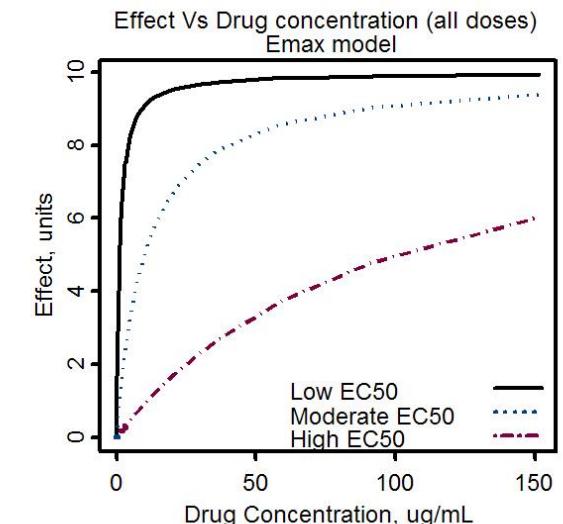
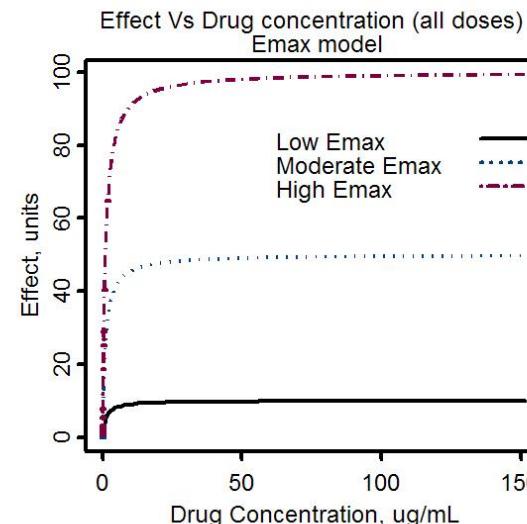
$E(t)$  : observed effect

$E_0$  : response without treatment (baseline)

$E_{max}$  : maximal effect

$C_{50}$  : concentration to reach  $E_{max}/2$

$C$  : drug concentrations



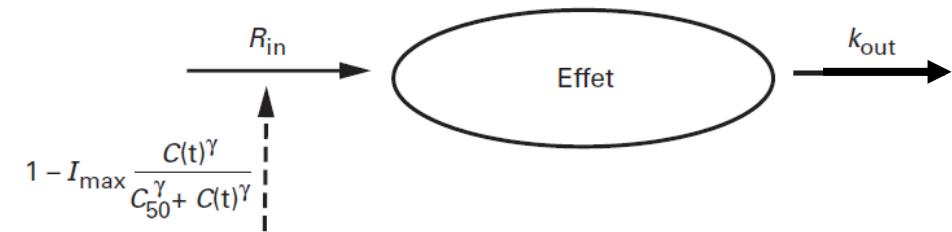
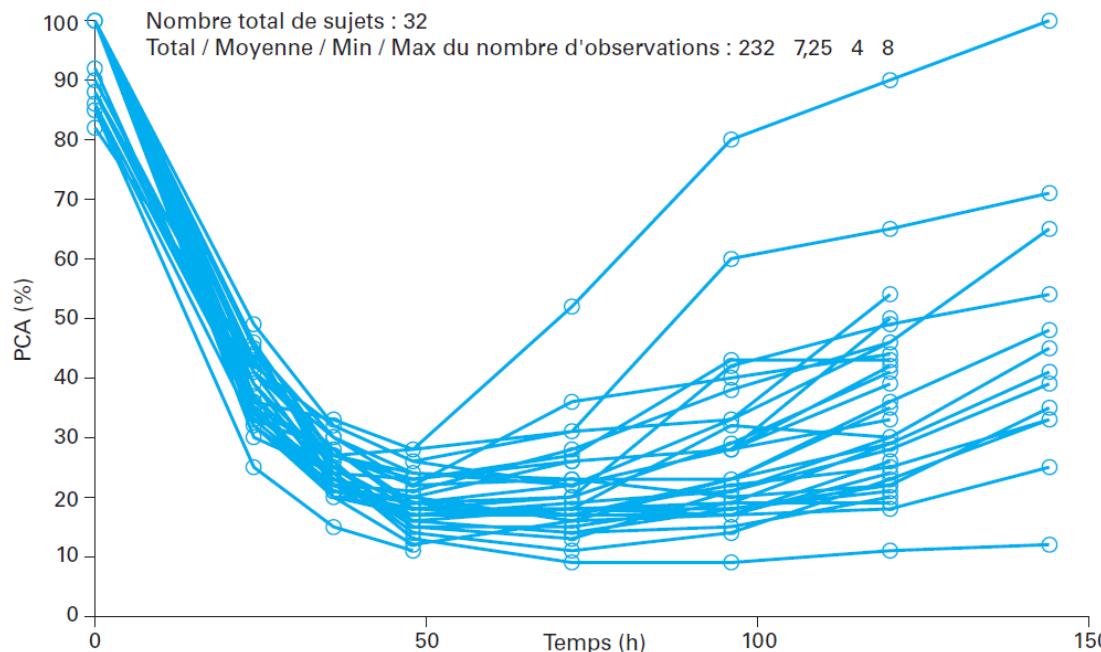
# PHARMACODYNAMIC MODEL

## Indirect response model

Lag between the drug action and the effect observed on the marker

Warfarin : inhibition of vitamin K recycling

- prevents formation of coagulation factors
- decrease of PCA (Prothrombin Complex Activity)



$$\frac{dE}{dt} = R_{in} \left( 1 - \frac{C(t)^{\gamma}}{C_{50}^{\gamma} + C(t)^{\gamma}} \right) - k_{out} E$$

$$E(t = 0) = \frac{R_{in}}{k_{out}}$$

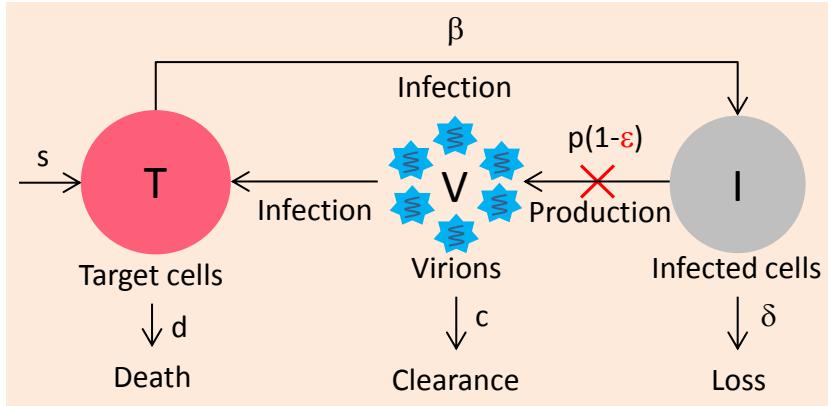
$R_{in}$  : Input (production of coagulation factors)  
 $k_{out}$  : Loss (degradation of coagulation factors)

# MODELS IN PHARMACOKINETICS AND PHARMACODYNAMICS

## PHARMACODYNAMIC MODEL

### Viral kinetic model

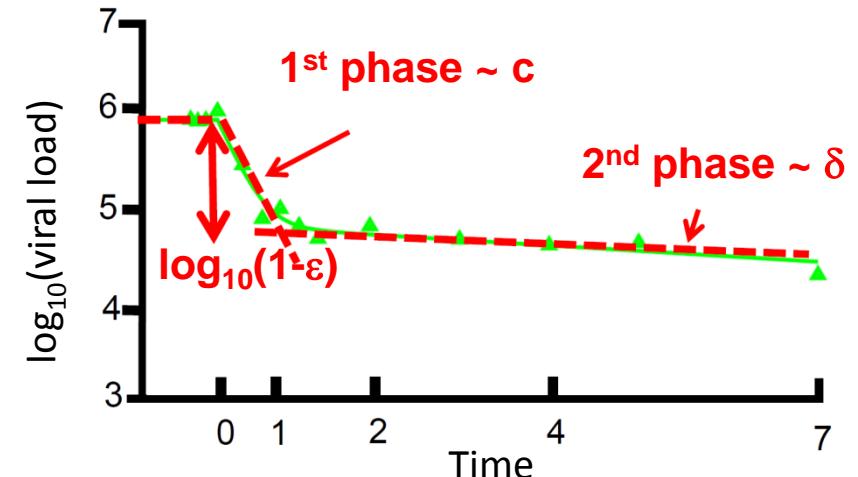
- A basic model (Neumann et al, Science. 1998)



$$\frac{dT}{dt} = s - dT - \beta VT$$

$$\frac{dI}{dt} = \beta VT - \delta I$$

$$\frac{dV}{dt} = (1 - \varepsilon)pI - cV$$



s: production rate of target cells

d: death rate of target cells

$\beta$ : infection rate

p: production rate per infected cell

c: clearance rate of free virus

$\delta$ : loss rate of infected cells

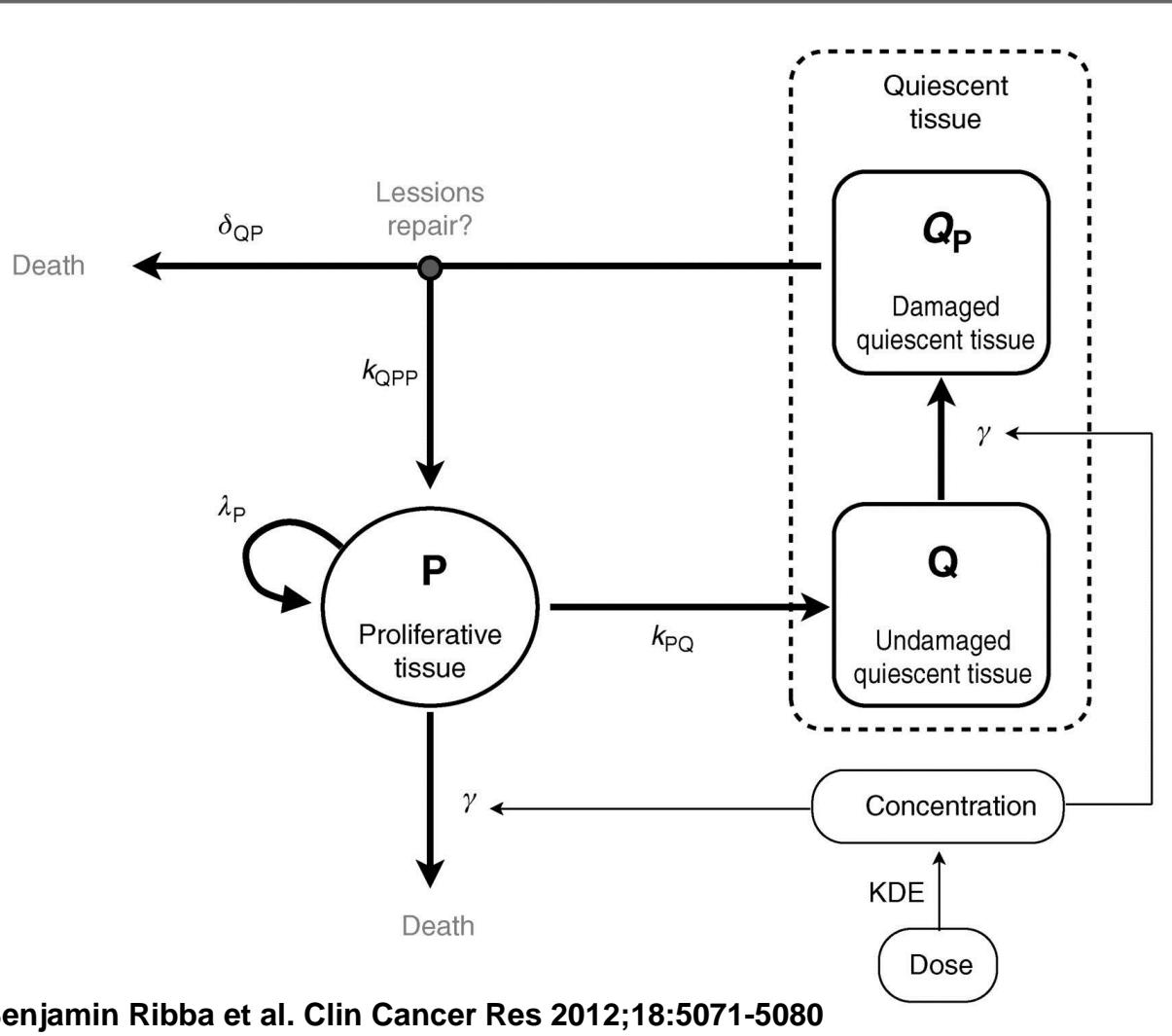
$\varepsilon$ : treatment effectiveness

# MODELS IN PHARMACOKINETICS AND PHARMACODYNAMICS

## PHARMACODYNAMIC MODEL

19

### Tumor growth inhibition model



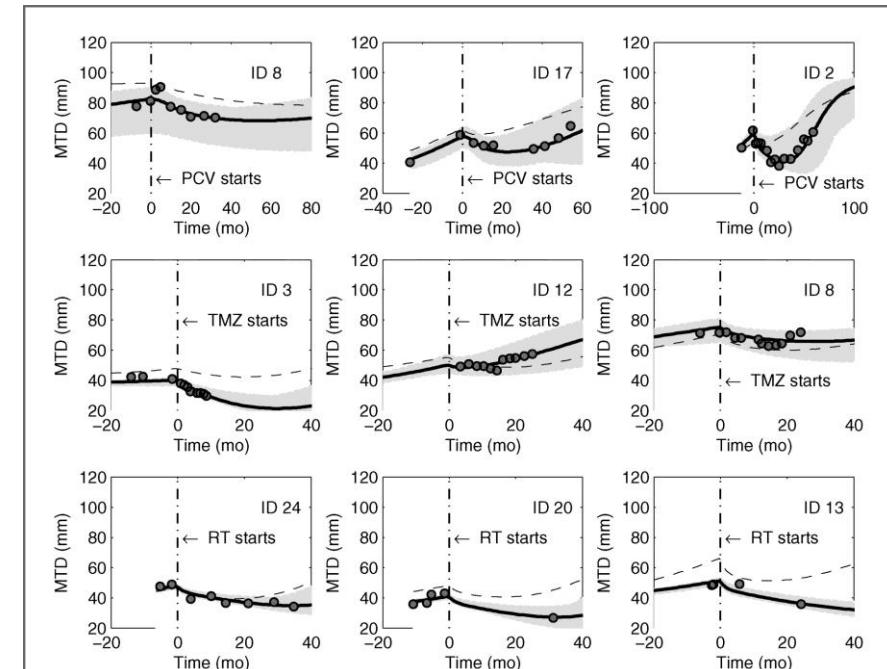
$$\frac{dC}{dt} = -\text{KDE} \times C$$

$$\frac{dP}{dt} = \lambda_p \times P \left(1 - \frac{P^*}{K}\right) + k_{Q_P P} \times Q_P - k_{PQ} \times P - \gamma_P \times C \times \text{KDE} \times P$$

$$\frac{dQ}{dt} = k_{PQ} P - \gamma_Q \times C \times \text{KDE} \times Q$$

$$\frac{dQ_P}{dt} = \gamma_Q \times C \times \text{KDE} \times Q - k_{Q_P P} Q_P - \delta_{QP} \times Q_P$$

$$P^* = P + Q + Q_P$$

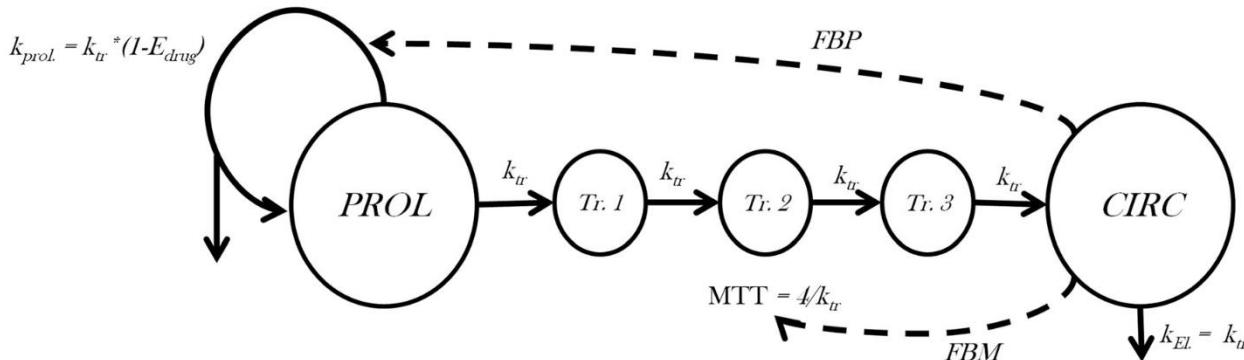


# MODELS IN PHARMACOKINETICS AND PHARMACODYNAMICS

## PHARMACODYNAMIC MODEL

20

### Drug-induced thrombocytopenia model



$$\frac{dPROL}{dt} = k_{prol} \cdot (1 - E_{drug}) \cdot FBP \cdot PROL - k_{tr} \cdot FBM \cdot PI$$

$$\frac{dTr\ 1}{dt} = k_{tr} \cdot FBM \cdot PROL - k_{tr} \cdot FBM \cdot Tr\ 1$$

$$\frac{dTr\ 2}{dt} = k_{tr} \cdot FBM \cdot Tr\ 1 - k_{tr} \cdot FBM \cdot Tr\ 2$$

$$\frac{dTr\ 3}{dt} = k_{tr} \cdot FBM \cdot Tr\ 2 - k_{tr} \cdot FBM \cdot Tr\ 3$$

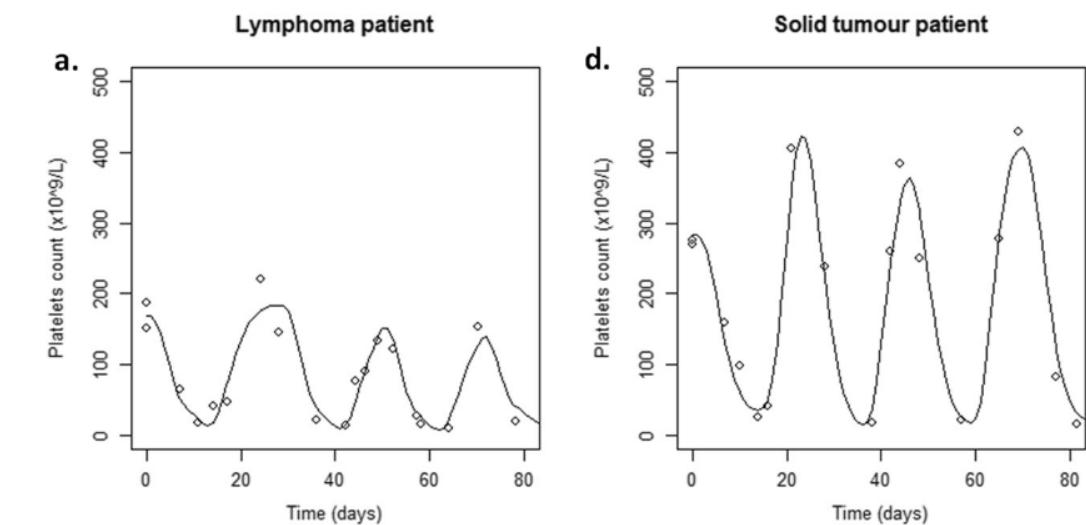
$$\frac{dCIRC}{dt} = k_{tr} \cdot FBM \cdot Tr\ 3 - k_{EL} \cdot CIRC$$

$$E_{drug} = \frac{IMAX \cdot Conc}{IC_{50} + Conc}$$

$$FBP = \left( \frac{BASE}{CIRC} \right)^{\gamma} \quad FBM = \left( \frac{BASE}{CIRC} \right)^{\delta}$$

If solid tumour patients :  $BASE = BASE_{0 ST}$

$$\text{If lymphoma patients : } BASE = BASE_{0 LY} - \frac{IMAT \cdot t}{IT_{50} + t}$$

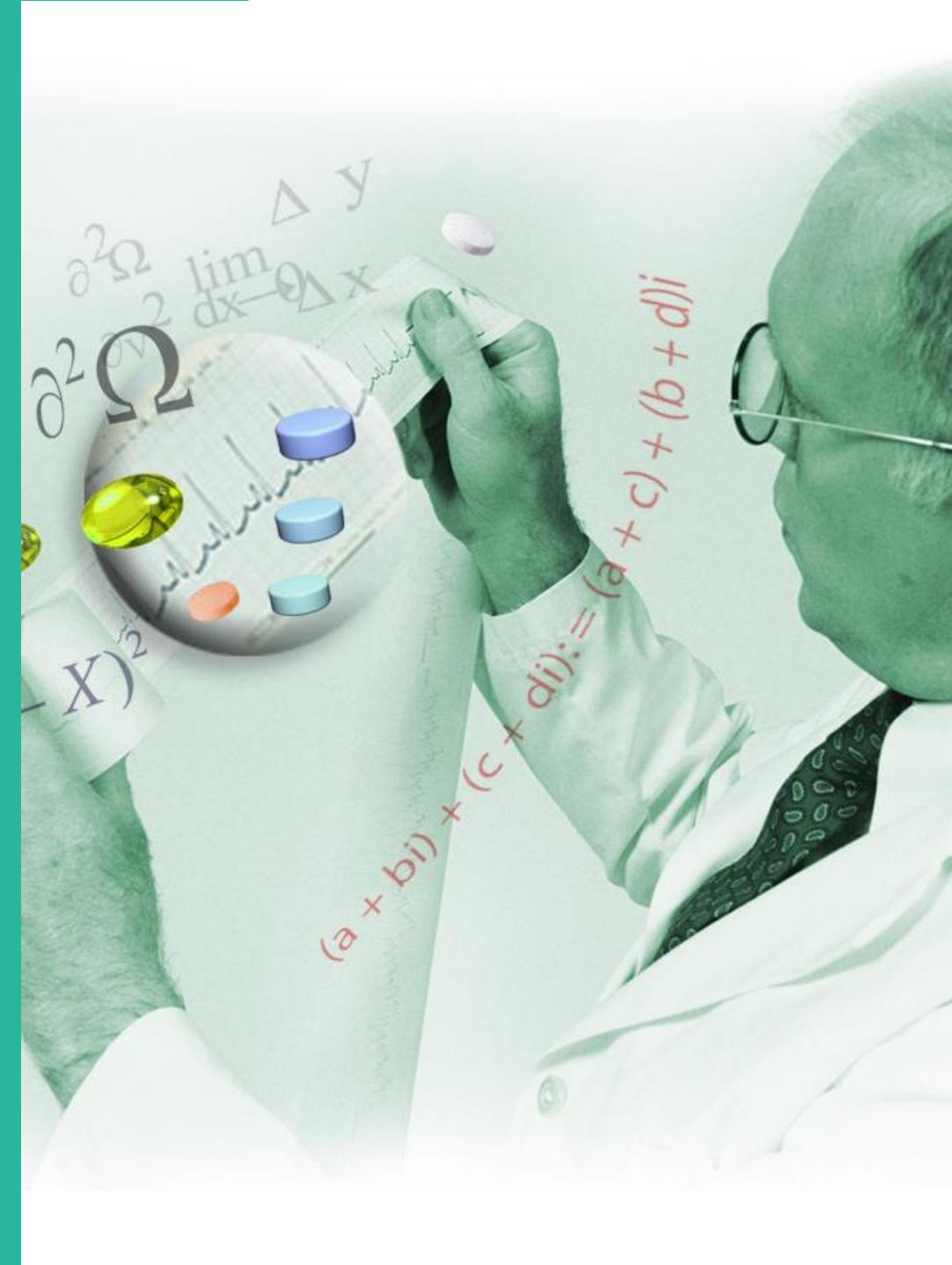


# INTERESTS FOR MODELING

- Quantitative summary of the evolution of profile across time through physiological parameters
- Better predictive / simulation ability for other doses, special populations...
- Analysis of all longitudinal data in clinical trials (not only the endpoint) : more powerful to detect drug effect and less bias through the inclusion of dropouts
- Test of hypothesis on effect mechanism of drugs
- Comparison of groups of patients through statistical comparison of parameters
- Statistical issues : nonlinear models, high interindividual variability
  - Nonlinear mixed effects models for parameters estimation

# STATICAL METHODS

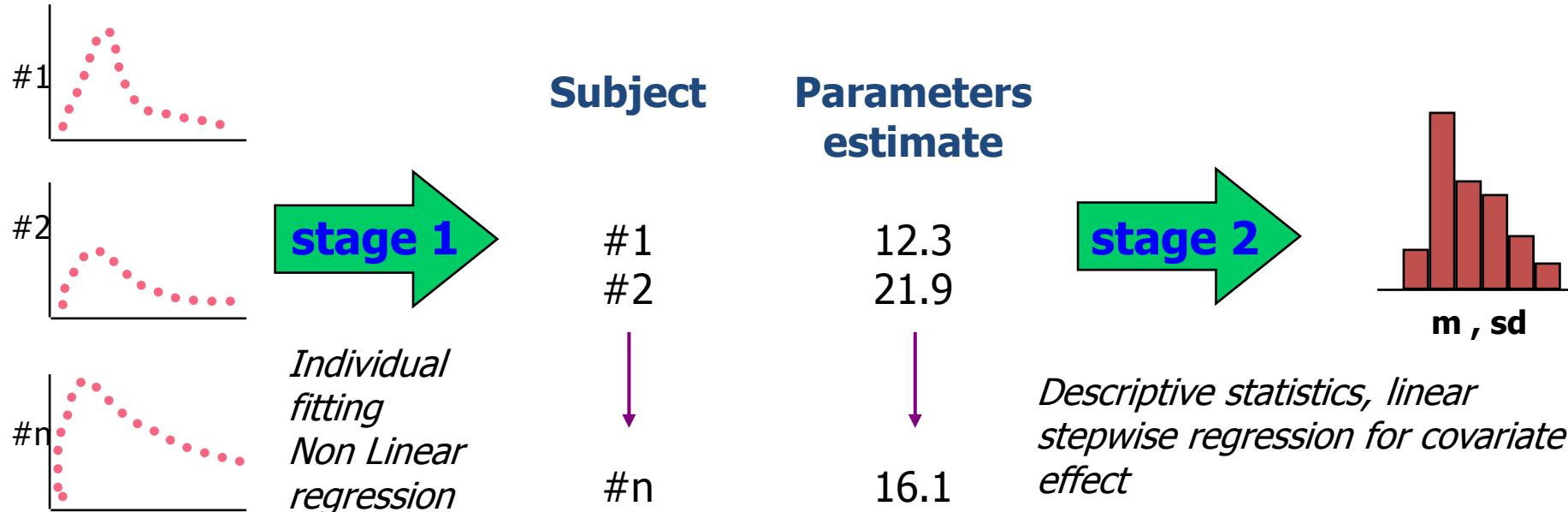
# PHARMACOMETRICS



Science of quantitative pharmacology

- Quantify the pharmacologic activity of a drug and its variability between subjects and/or between occasions
  - two-stage method
  - population approach
    - main tool : nonlinear mixed effects models

# TWO - STAGES METHOD



From Steimer (1992): « Population models and methods, with emphasis on pharmacokinetics », in M. Rowland and L. Aarons (eds), *New strategies in drug development and clinical evaluation, the population approach*

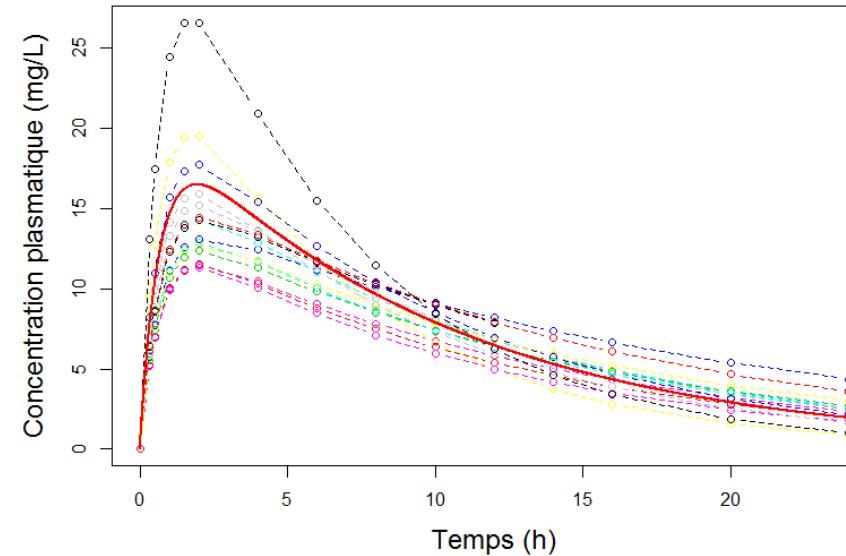
## 1. Individual nonlinear regression

- Estimation of individual parameters: require a large number of samples per subject

## 2. Statistical summary (mean, variance)

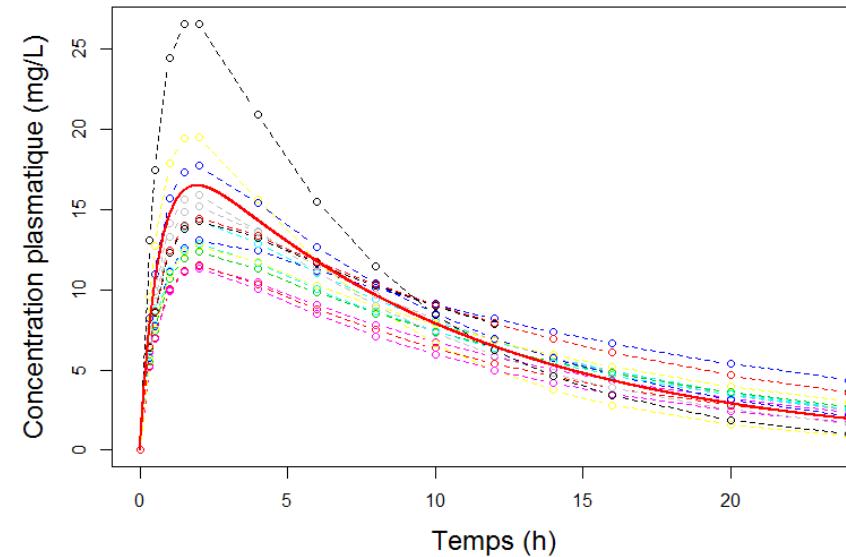
- Overestimate the variability (do not distinguish the variability between individuals of the residual error)

## 3. Relations with covariates (gender, weight...)



## Nonlinear mixed effects models

- Simultaneous analysis of all observations
  - reduce number of samples per subject
- Estimation of mean parameters and their variabilities (without bias)
- Identification of covariates influencing the variability
  - determination of relationships between covariates and model parameters



## Nonlinear mixed effects models

- Based on several statistical and mechanistic hypotheses
  - structural model (nonlinear function)
  - variability model
  - residual error model

# NONLINEAR MIXED EFFECTS MODELS

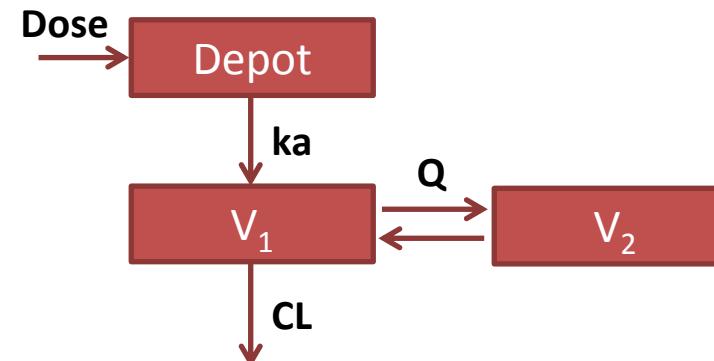
## Notations

Concentration  $y_{ij}$  for subject  $i$  observed at time  $t_{ij}$  :

$$y_{ij} = f(\theta_i, t_{ij}) + \varepsilon_{ij}$$

$f$  : structural model

- The same for all subjects
  - One equations system for all subjects
- a specific vector of parameters  $\theta_i = \{ka, V_1, Q, V_2, CL\}$  for subject  $i$ 
  - $\theta_i$  : individual parameters



$\varepsilon_{ij}$  : residual error

# NONLINEAR MIXED EFFECTS MODELS

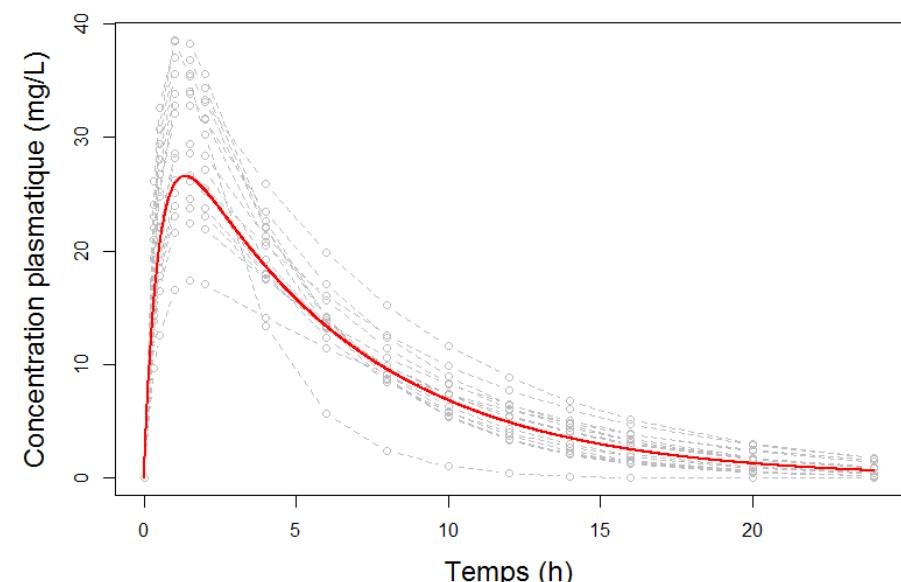
## Notations

$\theta_i$  : individual parameters

$$\theta_i = \mu e^{\eta_i}$$

$\mu$  : fixed effect (mean parameter)

- Estimated from observations of all subjects
- Mean profile predicted by integrating the mean values of parameters in the model ( $\{ka, V_1, Q, V_2, CL\}$ )



# NONLINEAR MIXED EFFECTS MODELS

## Notations

$\theta_i$  : individual parameters

$$\theta_i = \mu e^{\eta_i}$$

$\mu$  : fixed effect (mean parameter)

$\eta_i$  : random effects

- **hypothesis** : we assume the distribution of random effects is known

$$\eta_i \sim N(0, \omega^2)$$

➤  $\theta_i = \mu + \eta_i$ ,  $\theta_i$  follow a normal distribution

➤  $\theta_i = \mu e^{\eta_i}$ ,  $\theta_i$  follow a log-normal distribution ( $\theta_i > 0$ )

# NONLINEAR MIXED EFFECTS MODELS

## Notations

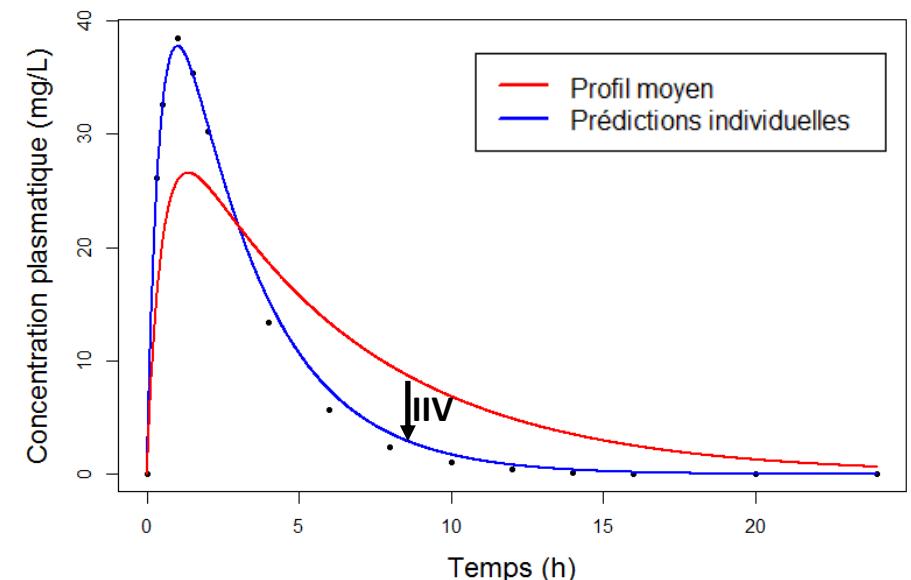
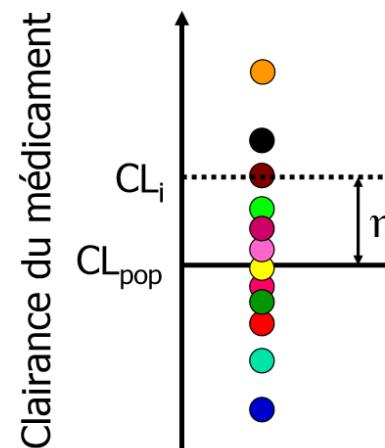
$\theta_i$  : individual parameters

$$\theta_i = \mu e^{\eta_i}$$

$\mu$  : fixed effect (mean parameter)

$\eta_i$  : random effects

- interindividual variability



# NONLINEAR MIXED EFFECTS MODELS

## Notations

$$y_{ij} = f(\theta_i, t_{ij}) + \varepsilon_{ij}$$

$\theta_i$  : individual parameters

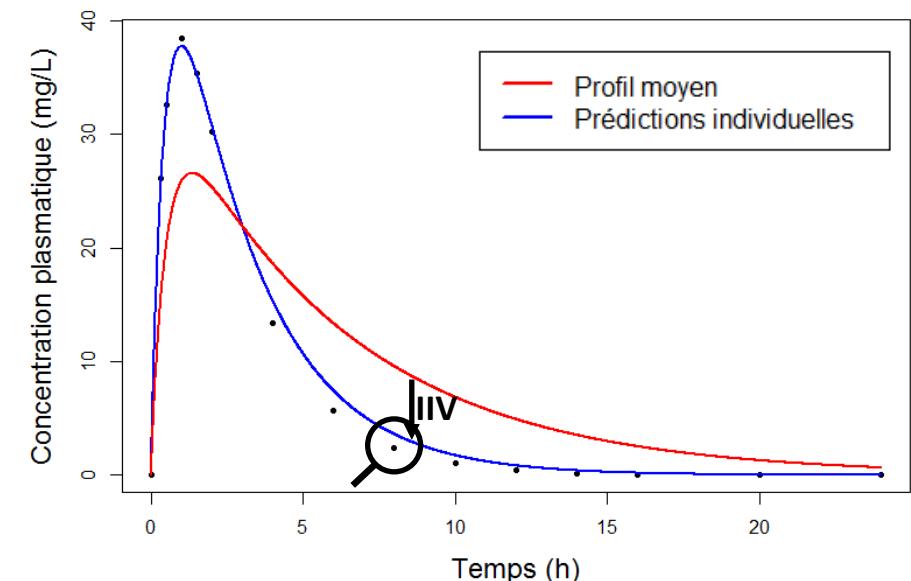
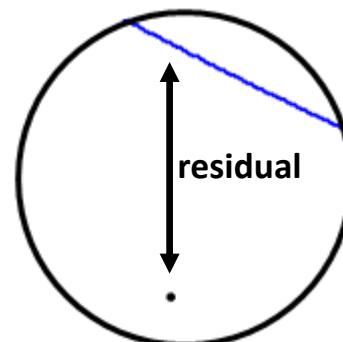
$$\theta_i = \mu e^{\eta_i}$$

$\mu$  : fixed effect (mean parameter)

$\eta_i$  : random effects

$\varepsilon_{ij}$  : residual error

- hypothesis :  $\varepsilon_{ij} \sim N(0, \sigma^2)$



# NONLINEAR MIXED EFFECTS MODELS

## Notations

$\theta_i$  : individual parameters

$$\theta_i = \mu e^{\eta_i}$$

$\mu$  : fixed effect (mean parameter)

$\eta_i$  : random effects,  $\eta_i \sim N(0, \omega^2)$

$\varepsilon_{ij}$  : residual error,  $\varepsilon_{ij} \sim N(0, \sigma^2)$

- Distinction between interindividual variability and residual error

Parameters to estimate :

$$\{\mu, \omega^2, \sigma^2\}$$

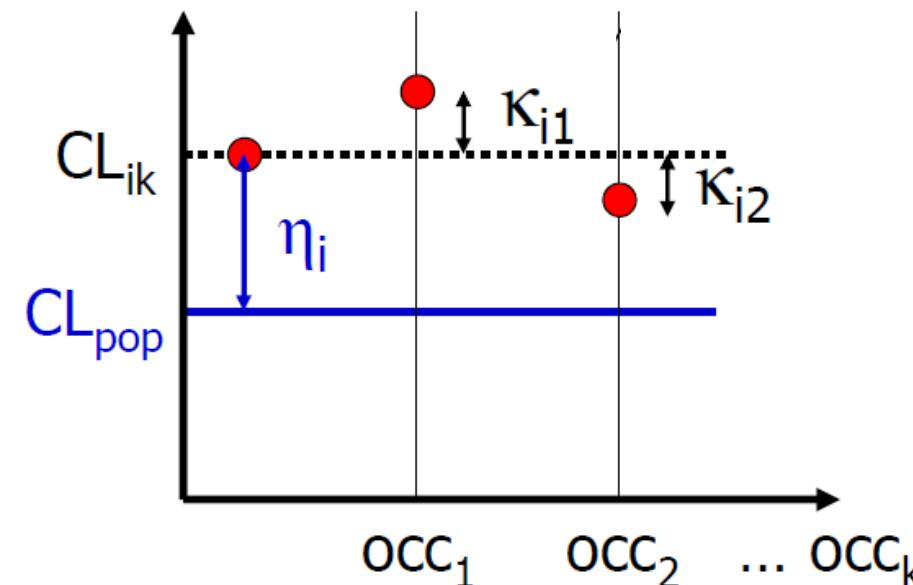
# NONLINEAR MIXED EFFECTS MODELS

## Intraindividual variability

$\kappa_i$  : interoccasion variability

- data collected at different periods
  - Different visits
  - Changing in treatment schedule, trial arm

$$\theta_i = \mu e^{\eta_i + \kappa_i}$$



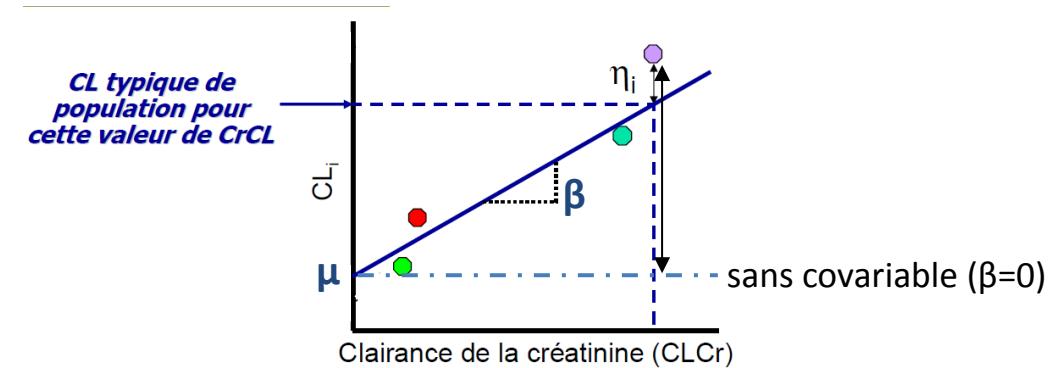
# NONLINEAR MIXED EFFECTS MODELS

## Covariates

- Physiological, biological, pharmacological specificities...
- Explain the sources of parameters variability

- Continuous covariates

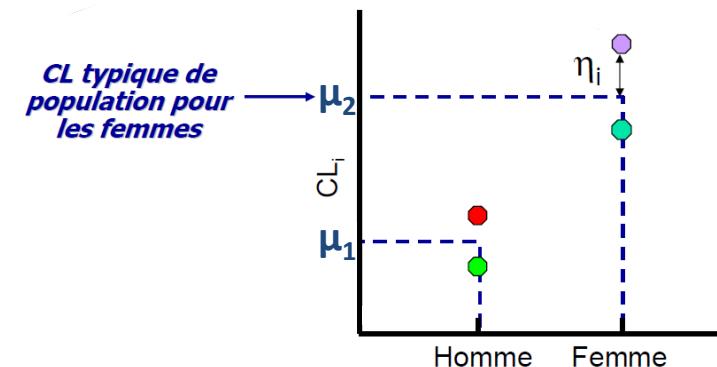
$$CL_i = \mu + \beta \times CLCr_i + \eta_i$$



- Binary covariates

$$CL_{i,homme} = \mu_1 + \eta_i$$

$$CL_{i,femme} = \mu_2 + \eta_i$$



# NONLINEAR MIXED EFFECTS MODELS

## Estimation

$$C(t) = \frac{F \times \text{Dose}}{V} \frac{ka}{(ka - \frac{CL}{V})} \left( e^{-\frac{CL}{V}t} - e^{-ka t} \right)$$

$$\theta_F = \mu_F e^{\eta_{Fi}}, \eta_{Fi} \sim N(0, \omega^2_F)$$

$$\theta_{ka} = \mu_{ka} e^{\eta_{ka_i}}, \eta_{ka_i} \sim N(0, \omega^2_{ka})$$

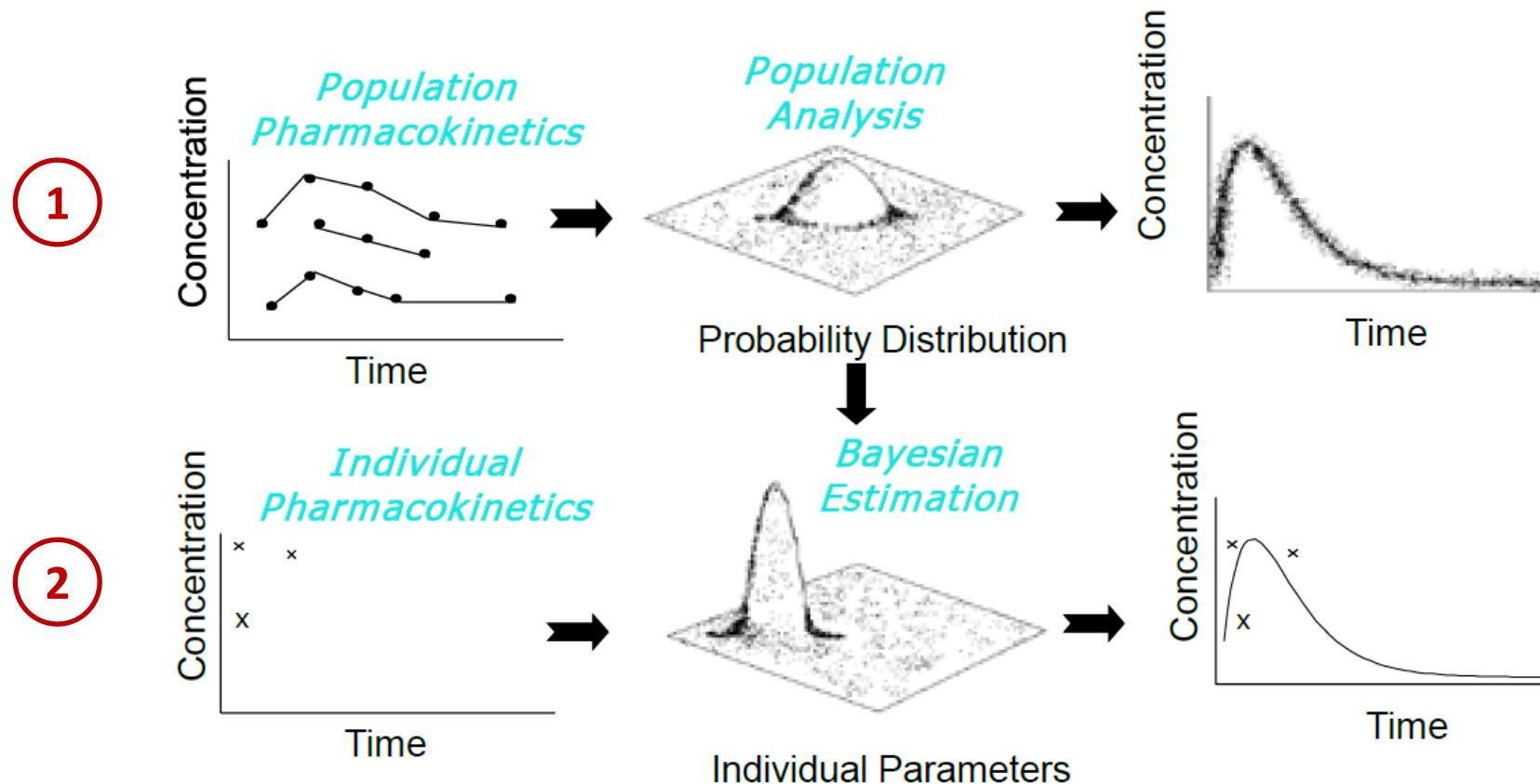
$$\theta_V = \mu_V e^{\eta_{Vi}}, \eta_{Vi} \sim N(0, \omega^2_V)$$

$$\theta_{CL} = \mu_{CL} e^{\eta_{CL_i}}, \eta_{CL_i} \sim N(0, \omega^2_{CL})$$

$$\varepsilon_{ij} \sim N(0, \sigma^2)$$

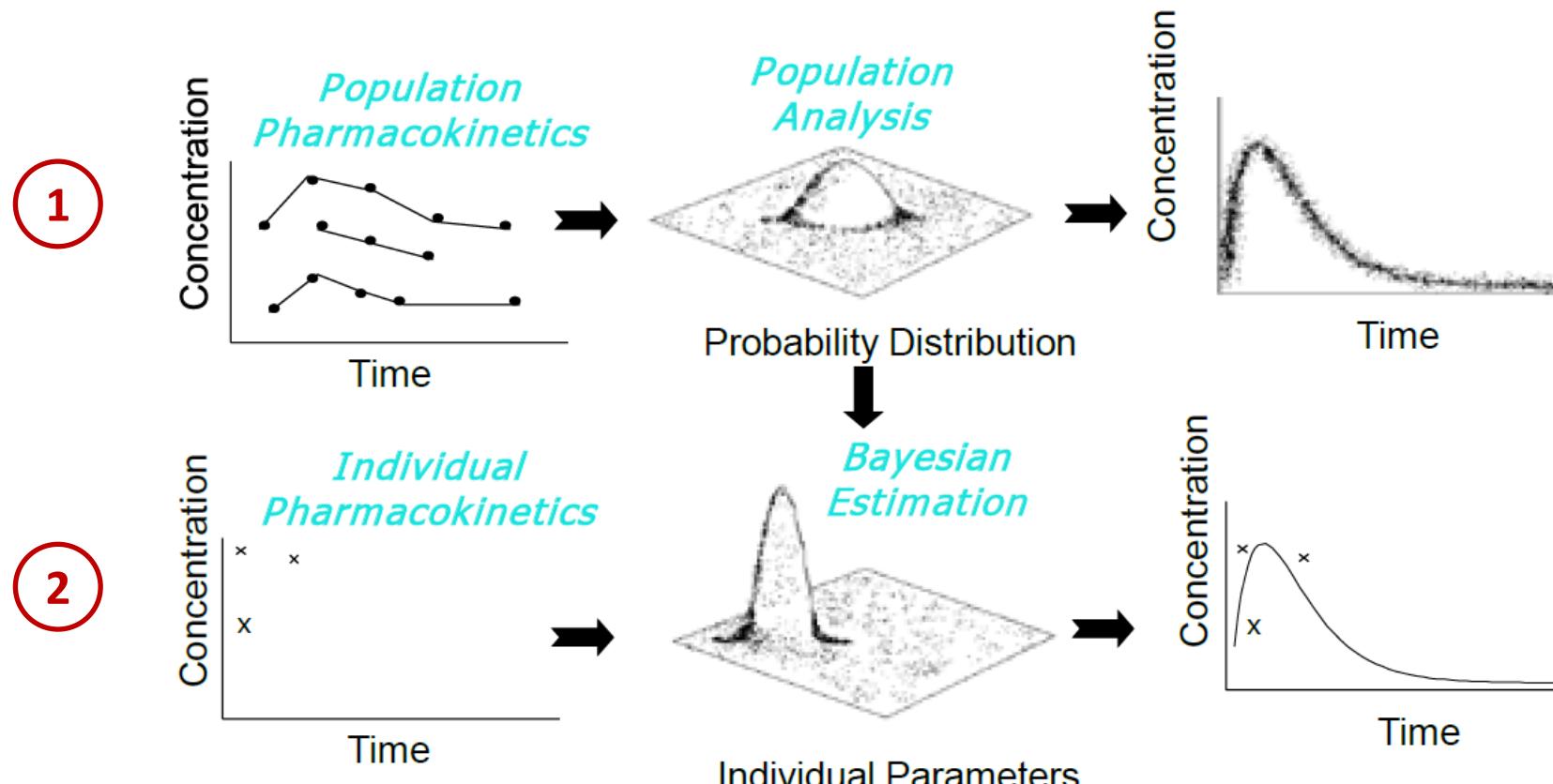
- Estimation of fixed and random effects?

# NONLINEAR MIXED EFFECTS MODELS



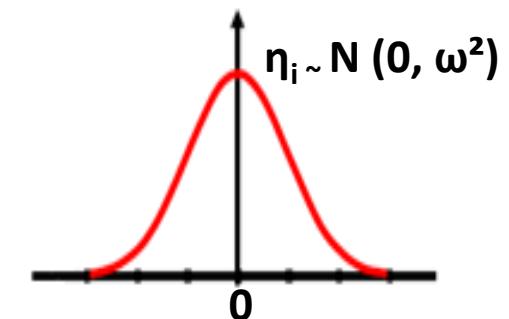
Steimer JL, Vozeh S, Racine Poon A, Holford N, O'Neil R: The population approach: rationale, methods and applications in clinical pharmacology and drug development. In P.G. Welling & L. Balant (eds), *Handbook of experimental pharmacology* (vol 110 : Pharmacokinetics of drugs, Berlin : SpringVerlag, 1994, 405-451)

# NONLINEAR MIXED EFFECTS MODELS

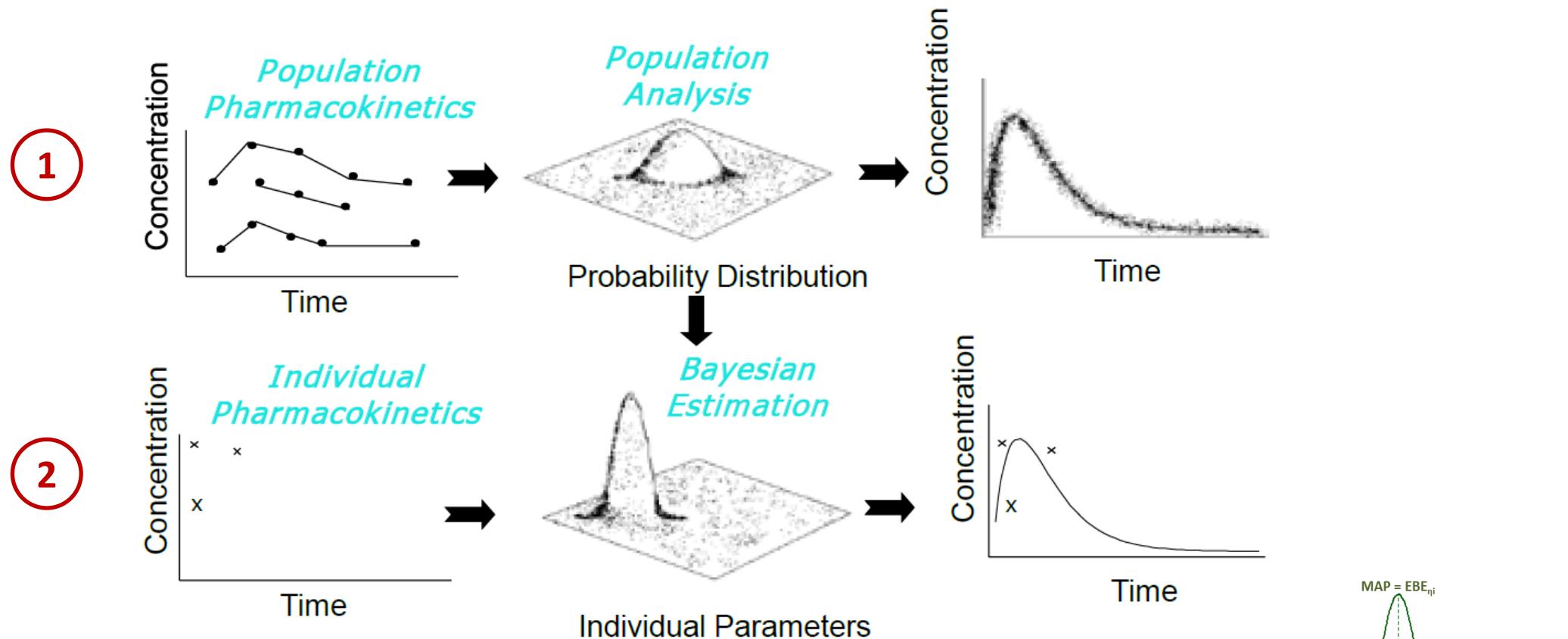


## 1. Analysis of all observations

- Estimation of population parameters ( $\mu, \omega^2, \sigma^2$ )
  - maximum likelihood
  - prior distribution

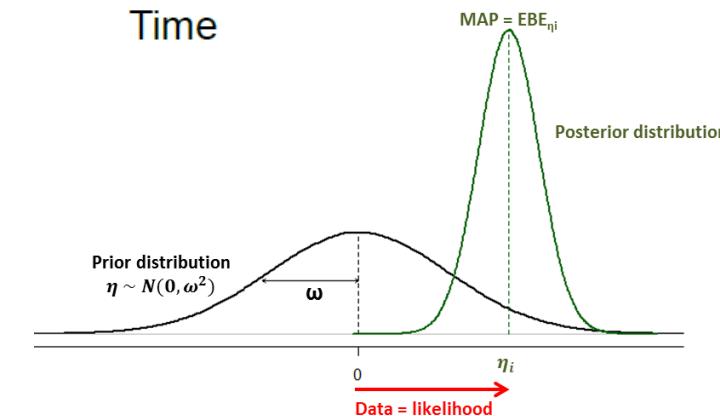


# NONLINEAR MIXED EFFECTS MODELS



## 2. Estimation of individual parameters

- Bayesian method
  - maximum a posteriori = individual parameters
  - $\theta_i = \mu e^{\eta_i}$



# NONLINEAR MIXED EFFECTS MODELS

## Likelihood

$$L(\theta, y) = p(y|\theta)$$

- Probability to observe  $y$  knowing  $\theta$
- Maximum likelihood : estimate the parameters  $\theta$  for model predictions are as close as possible to the observed data

## Issue

- $f$  is nonlinear in its parameters
  - no analytical expression of the likelihood
  - required to approximate the likelihood
    - estimation algorithms

# NONLINEAR MIXED EFFECTS MODELS

## First estimation method

**NON linear Mixed Effects Model**  
L. Sheiner & S. Beal, UCSF

- **1972: Concept and FO method**

Sheiner, L. B., Rosenberg, B. & Melmon, K. L. Modelling of individual pharmacokinetics for computer-aided drug dosage. *Comput. Biomed. Res. Int. J.* **5**, 411–459 (1972).

- **1977: First publication**

Sheiner, L. B., Rosenberg, B. & Marathe, V. V. Estimation of population characteristics of pharmacokinetic parameters from routine clinical data. *J. Pharmacokinet. Biopharm.* **5**, 445–479 (1977).

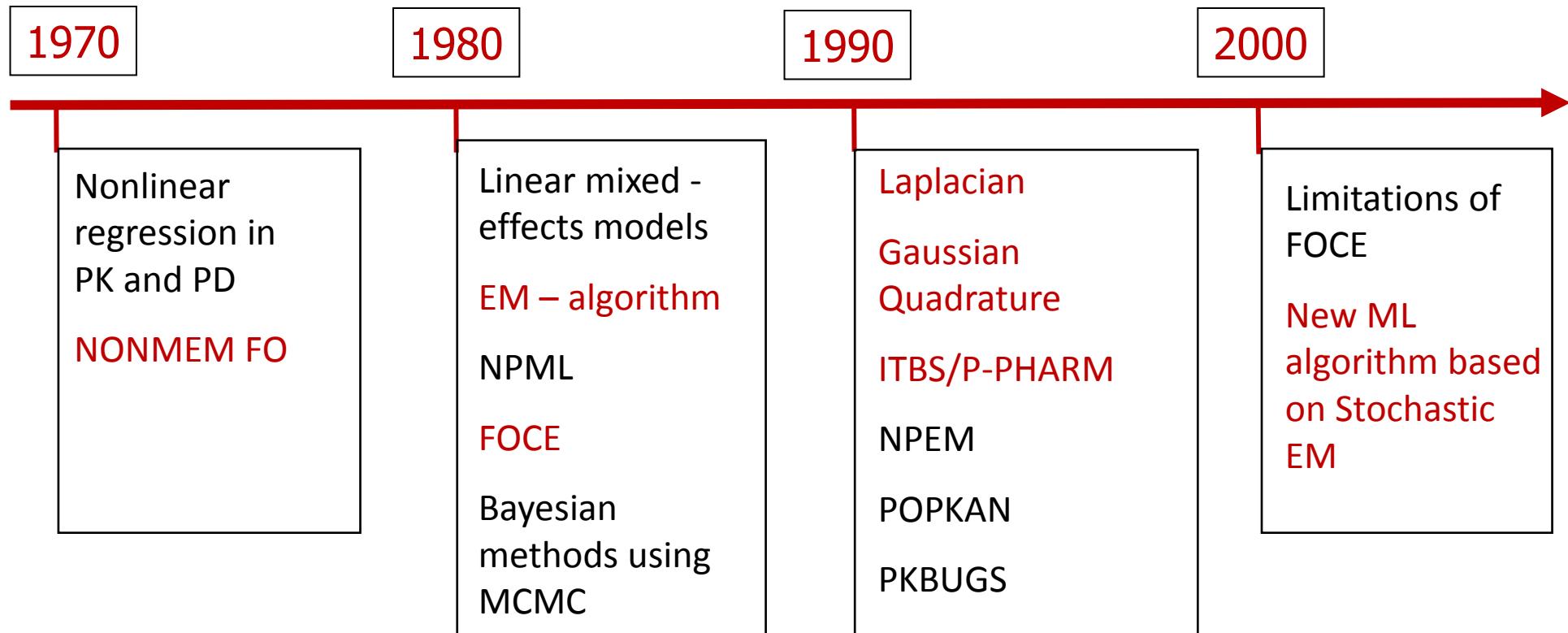
- **1980: NONMEM - first software**

Beal, S. L. & Sheiner, L. B. The NONMEM system. *Am Stat.* **34**, 118–119 (1980).

Beal, S. L. & Sheiner, L. B. Estimating population kinetics. *Crit. Rev. Biomed. Eng.* **8**, 195–222 (1982).

# NONLINEAR MIXED EFFECTS MODELS

## Development of estimation methods



Pillai, G. C., Mentré, F. & Steimer, J.-L. Non-linear mixed effects modeling - from methodology and software development to driving implementation in drug development science. *J. Pharmacokinet. Pharmacodyn.* **32**, 161–183 (2005).

# NONLINEAR MIXED EFFECTS MODELS

## Estimation softwares for nonlinear mixed effects models

**Tableau 2 – Logiciels de population les plus utilisés**

Logiciel	Algorithmes disponibles	Type	Interface
Monolix	SAEM	Mixte (1)	Oui
NONMEM	FO, FOCE, Laplace, SAEM, Bayes	Payant	Non
Phoenix	FOCE	Payant	Oui
R (librairies)	nlme, lme4 (FOCE, Laplace), saemix (SAEM)	Gratuit	Non (2)
SAS	NLMIXED (FO, FOCE, AGQ)	Payant	Oui
WinBugs	Bayes	Gratuit	Non

(1) Licence gratuite pour les universitaires et étudiants.

(2) R n'a pas d'interface graphique spécifique mais des outils comme Rstudio peuvent être utilisés pour le faire tourner.

# NONLINEAR MIXED EFFECTS MODELS

## Model development

Find the model which describes adequately the data, by determining :

- Structural model
- Variability model (inter and intraindividual)
- Residual error model
- Covariates

No consensus on building method

- Development of a basic model without covariates
- Analysis and integration of significant covariates in model

# NONLINEAR MIXED EFFECTS MODELS

## Model selection

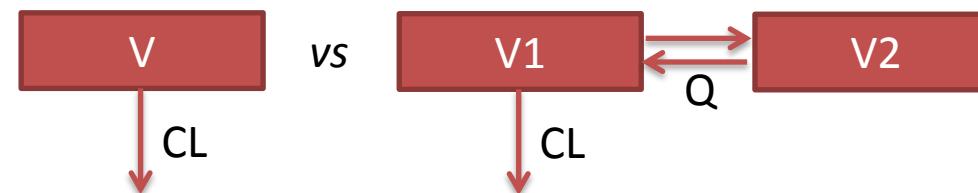
Parsimony : the model which best describe the data with the lower number of parameters

Statistical criteria:

- $-2LL = -2 \log(\text{likelihood})$ 
  - approximate of likelihood to minimize
- Other criteria: AIC, BIC

Likelihood Ratio Test (LRT)

- Reduced model ( $p$  parameters) :  $-2LL_{reduced}$
- Full model ( $p + q$  parameters) :  $-2LL_{full}$
- Under  $H_0$  :  $-2LL_{reduced} - -2LL_{full} \sim \chi^2$  ( $ddl = q$ )



# NONLINEAR MIXED EFFECTS MODELS

## Model evaluation

### Estimation precision

$$RSE (\%) = \frac{\text{Standard Error}}{\text{Parameter estimate}}$$

### Graphical evaluation

- Comparison of model predictions to observed data
- Residuals evaluation
- Simulations based evaluation
  - VPC (Visual Predictive Check)
  - NPDE (Normalized prediction distribution errors)

### Numerical evaluation

- data splitting
- bootstrap
- Jack-knife

---

# TREATMENT INDIVIDUALIZATION

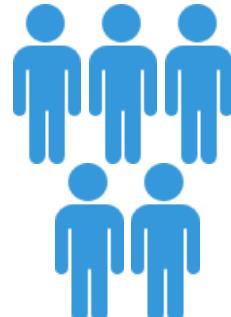


PHOTOGRAPH BY ADAM VOORHES

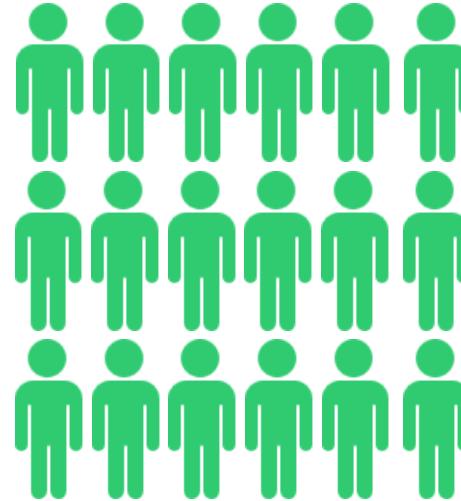
# TREATMENT INDIVIDUALIZATION THERAPEUTIC INTERESTS

48

Same diagnosis, same prescription



Drug toxic  
but beneficial



Drug NOT toxic  
and beneficial



Drug NOT toxic  
and NOT beneficial



Drug toxic  
and NOT beneficial

- Administer the right dose for each patient depending on its features and characteristics of the drug (therapeutic range)

## Methods

- *a priori* adaptation
- *a posteriori* adaptation through Bayesian method

# TREATMENT INDIVIDUALIZATION *A PRIORI* ADAPTATION

49

Model : structure, variability, covariates

Patient : no PK data, only subject characteristics (age, weight, biology...)

- Predict the patient PK parameters using model and covariates values
  - Predict concentrations for this subject
  - Limited when variability is high or with a limited number of covariates in model

Prediction of carboplatin clearance :

- 4 covariates associated to carboplatin clearance

$$CL = \mathbf{0.134} \times \mathbf{weight} + \frac{\mathbf{218} \times \mathbf{weight} \times (1 - \mathbf{0.00457} \times \mathbf{age}) \times (1 - \mathbf{0.314} \times \mathbf{sex})}{\mathbf{serum \ creatinine}}$$

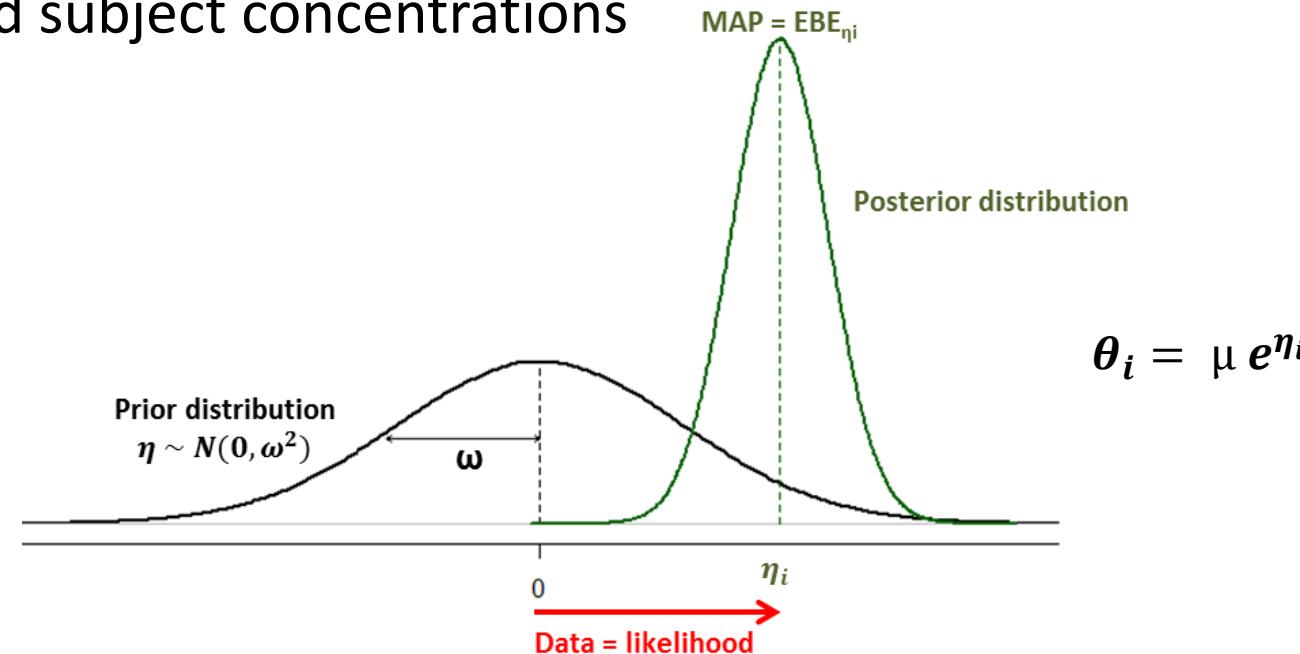
$AUC = Dose/CL$  : determine the dose to reach the targeted l'AUC?

Chatelut, E. et al. Prediction of carboplatin clearance from standard morphological and biological patient characteristics. *J. Natl. Cancer Inst.* **87**, 573–580 (1995).

# A POSTERIORI ADAPTATION : BAYESIAN APPROACH

$$prior \propto data = posterior\ distribution$$

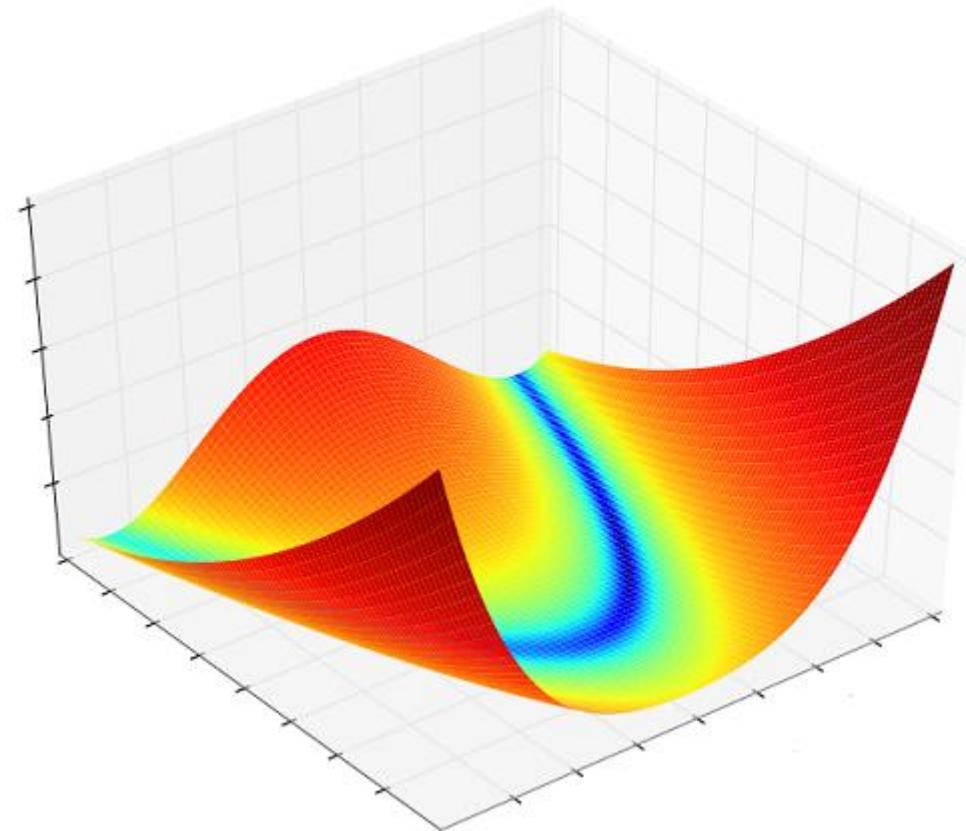
- Prior : population model parameters (mean  $\mu$  and variance  $\omega^2$ )
- Data : measured subject concentrations



- Posterior distribution : individual PK parameters
  - Prediction of next concentrations
    - determine the next dose
    - periodic evaluation to optimize dose on intraindividual variability

---

# DESIGN OPTIMIZATION



# DESIGN FOR POPULATION PK-PD ANALYSIS

## Importance of the choice

- Influence the precision of parameters estimation
- Poor design can lead to unreliable studies
- All the more important in pediatric studies
  - severe limitations on the number of samples to be taken
  - ethical and physiological reasons

## Problem : choice of population design

- number of patients?
- number of sampling times?
- sampling times?

## Recommendations on design in the FDA guidance

# EVALUATION OF POPULATION DESIGN

Two approaches

- simulation studies: cumbersome!
- methodology based on the Fisher Information matrix (FIM) in NLMEM

Expression of MF for population PK

- complex
- based on a linearisation of the model around the fixed effects  
(Mentré, Mallet & Baccar. Biometrika, 1997) (Retout, Mentré & Bruno. Stat Med, 2002)

Principle

- to compute  $M_F$  and its inverse for each population design to be evaluated
  - from the population model
  - from a priori value of the population parameters
- expected standard errors on the parameters = root mean square of the diagonal of  $M_F^{-1}$

# OPTIMIZATION OF POPULATION DESIGN

## Design comparisons

- objective : to have the “smallest”  $M_F^{-1}$  or the “largest”  $M_F$
- criteria for matrix comparison
  - D-optimality, the most usual one:  $\det(M_F)$

## Optimization of exact or statistical designs

- Maximization of  $\det(M_F)$ 
  - Find the best design for a given value of the population parameters
- Optimization of both the sampling times and the group structure
  - Fedorov-Wynn (specific algorithm), Simplex algorithm..

# OPTIMIZATION OF POPULATION DESIGN

PFIM software

[www.pfim.biostat.fr](http://www.pfim.biostat.fr)

\*\*\*\*\* OPTIMISED DESIGN \*\*\*\*\*

Optimised population design:

Sample times for response: A

	prot.opti	subjects.opti	Subjects
1	c(0.5, 12, 24, 144)	0.6768466	21.65909
2	c(0.5, 24, 120, 144)	0.3231534	10.34091

Sample times for response: B

	prot.opti	subjects.opti	Subjects
1	c(0.5, 12, 24, 144)	0.6768466	21.65909
2	c(0.5, 24, 120, 144)	0.3231534	10.34091

Associated optimised criterion: 580.1989

\*\*\*\*\* EXPECTED STANDARD ERRORS \*\*\*\*\*

----- Fixed Effects Parameters -----

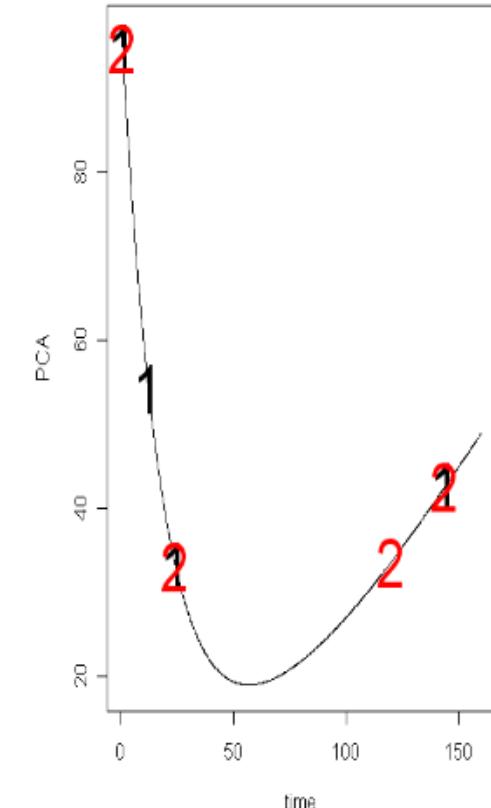
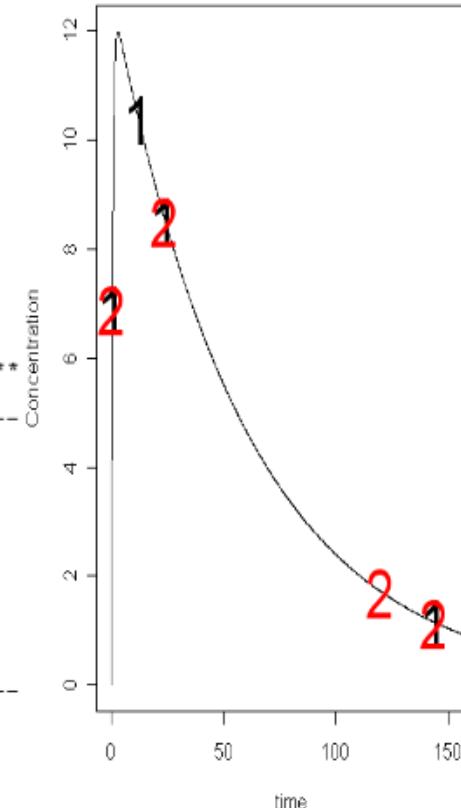
Beta	StdError	CV .	
Ka	1.600	0.263353095	16.459568 %
CL	0.133	0.006533504	4.912409 %
V	7.950	0.322403263	4.055387 %
Rin	5.410	0.437881955	8.093936 %
C50	1.200	0.052867047	4.405587 %
Kout	0.056	0.001737771	3.103163 %

----- Variance of Random Effects -----

Omega	StdError	CV .	
Ka	0.7010	0.206505767	29.45874 %
CL	0.0634	0.017561742	27.69991 %
V	0.0206	0.012226360	59.35126 %
Rin	0.1900	0.050298864	26.47309 %
C50	0.0129	0.016460059	127.59736 %
Kout	0.0167	0.007665362	45.90037 %

----- Variance of residual error -----

SIG	StdError	CV .	
sig.slopeA	0.20	0.0216894	10.84470 %
sig.interB	3.88	0.4677695	12.05591 %



→ Two groups with 22 and 10 subjects  
 → Total of 256 sampling times

---

# CONCLUSION



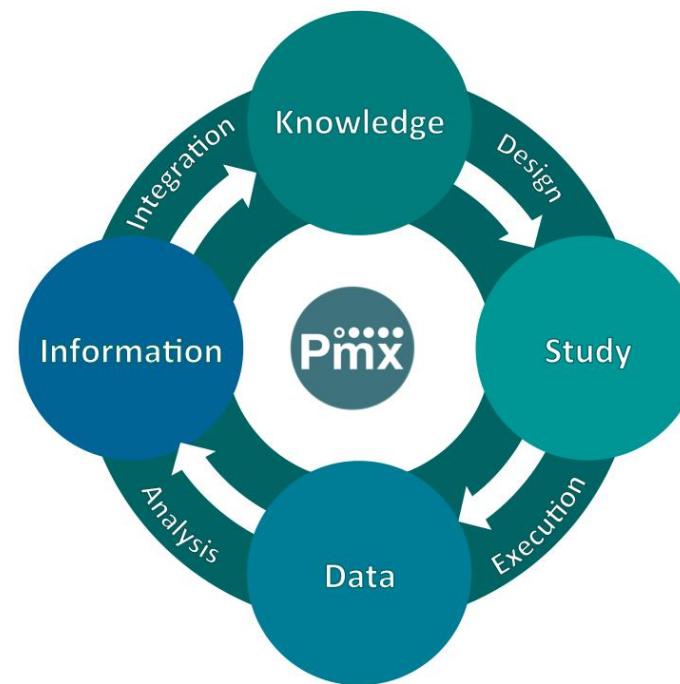
PHOTOGRAPH BY ADAM VOORHES

# INTERESTS FOR PHARMACOMETRICS

- Empirical or mechanistic description of data and PK-PD relationships
  - Now common method in the drug authorization application files
- Analysis of pharmacodynamic data ( $E_{max}$ ,  $EC_{50}$ )
- Analysis of sparse data (phase II and phase III)
- Estimation of variability and sources of variability (covariates)
- Prediction : other dosage schemes (schedule, dose, administration route...), sub-population (children, renal impairments...)
- Planning of next studies (clinical trial simulations, optimal design)
- Treatment individualization

## Model-Based Drug Development

- Guiding the drugs development through the use of pharmacometrics



# CONCLUSION FINALLY

59

- Increasing role of quantitative analysis of data through models in drugs evaluation
- Cooperative work
  - biologists, pharmacologists, clinicians
  - engineers, mathematicians, statisticians
- Many open methodological problems...

} Pharmacometrists

Thank you for your attention